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Housing Prices and Credit Constraints in Competitive Search*

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Abstract

In this paper we embed a directed search model of the real estate market into a heterogeneous agents setting to study the effect of credit on housing prices. Households can either rent or own their home and face idiosyncratic turnover shocks which make them want to change residence. They can accumulate financial assets to put a down payment on a home and to smooth consumption. Search and matching frictions generate frictional dispersion in housing prices and financial assets in equilibrium. Our model is "block recursive" and highly tractable. We calibrate it to reproduce selected statistics for the US. We extend the Endogenous Grid Method with non-convexities to our environment to compute it. In our framework the distribution of wealth, housing prices, and trading probabilities (e.g. liquidity of housing assets) are crucially affected by credit conditions. Our mechanism greatly amplifies the effect of changes in financial conditions on housing prices.

Keywords: wealth inequality, incomplete markets, directed search, housing prices, price dispersion, block-recursivity, endogenous grid method.
JEL Classification: D31, D83, E21, R21, R30.

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1 Introduction

In the expansion period prior to the Great Recession housing prices experienced significant increases at the same time that financial conditions eased greatly. There is a general consensus that the increase in the availability of credit fueled the housing boom; see, for instance, Glaeser et al. (2012), Landvoigt et al. (2015), and Favara and Imbs (2015). Yet the heterogeneous agents literature has struggled to explain the large impact of the increase in credit on housing prices. The reason is that, typically, constrained agents are poor and constitute a small fraction of the population, so they cannot really affect prices; see, for instance, Kiyotaki et al. (2011) and Sommer et al. (2013). More recent work by Favilukis et al. (2017) shows that a heterogeneous agents framework which combines aggregate risk and rich idiosyncratic heterogeneity can deliver a significant quantitative impact of credit conditions on prices.

Search and matching models, however, constitute a powerful mechanism for demand shocks to affect aggregates; see, for instance, Díaz and Jerez (2013). Also, a recent but growing number of quantitative studies uses these models to account for several dynamic and cyclical features of housing markets.[1] Yet most of this literature assumes that households are risk neutral and ignores the households' savings decisions.[2] But, for credit constraints to matter, we need to assume that households are risk averse and can accumulate financial assets. This paper thus embeds a search-theoretic model of the housing market into a heterogeneous agents setting to study the effect of credit on housing prices. We focus our analysis on stationary equilibria.

We consider an environment where households face idiosyncratic turnover shocks which make them want to change residence. Households consume a nondurable good and housing services, and can either own or rent their home. Home buyers must search the real estate market in order to buy a home, the search process being directed as in Moen (1997) and Acemoglu and Shimer (1999). Households can also accumulate a risk-free asset which can be used both to smooth consumption of the non-housing good and to put a down payment on a home. To keep things simple, we assume a fixed stock of owner-occupied housing which consists of symmetric (indivisible) units. In particular, all price dispersion arising in equilibrium is then purely *frictional*.

[1] See Díaz and Jerez (2013), Ngai and Tenreyro (2014), Head et al. (2014), and Hedlund (2016b), among others.

[2] Eerola and Maattanen (2018) and Hedlund (2016a 2016b) (as well as more recent work by this author with others) are notable exceptions.

We show that, in equilibrium, home buyers with higher financial wealth –who have a lower marginal utility of wealth– direct their search to submarkets with higher prices and shorter average buying times (relative to poorer buyers). In turn, homes with higher prices take longer to sell.[3] While frictional price dispersion is hard to measure —houses being hedonic goods, which are heterogeneous along many dimensions (some of which are unobservable)— several hedonic analysis in the real estate literature find variations in house prices after controlling for house characteristics and location using data for different countries (e. g. see Malpezzi et al., 1980; Elder et al., 1999; Leung et al., 2006; Yiu et al., 2008; Qiu and Tu, 2018).[4] In our setting, any shock affecting demand affects the distribution of prices paid by home buyers, and thus the mean and variance of housing prices. It also affects the trading delays faced by different buyers and the degree of housing market liquidity (e.g. as measured by average time on the market).

We calibrate our model to reproduce selected statistics of the Survey of Consumer Finances for working age households, such as the homeownership rate, the median wealth to earnings ratio for renters, the median housing wealth to earnings ratio for owners and their median Loan-To-Value ratio. Additionally, we calibrate the meeting technology to match the observed median time to buy reported by the National Association of Realtors.

Our quantitative exercises show that wealth accumulation, price dispersion and market liquidity are tightly linked, and that the interaction between these variables is crucially affected by credit conditions. Take the case of a highly liquid market, where demand is high and average buying (selling) times are long (short). In this scenario buyers who do not find a trading opportunity (a likely event for poor households) accumulate more assets and, in the next period, they direct their search towards a submarket with a higher price, where they are more likely to trade. Hence, when credit is eased, these buyers can afford to enter submarkets with higher prices. Not only that, but buyer participation also increases, as those other households who were not searching for a home (e.g. because they were waiting to build a down payment) access the market. This further increases buyer demand and the overall congestion all buyers face, thus reinforcing the aforementioned asset accumulation process by households. This mechanism, which operates through the inherent hetero-

[3] These results are consistent with empirical work in the real estate literature which finds that, after controlling for housing attributes and location, buyers with higher income tend to pay higher prices (see Elder et al., 1999; Qiu and Tu, 2018), and search for a shorter period of time on average (see Elder, Zumpano, and Baryla 1999, 2000). There is also widespread evidence of a positive relation between the price of real estate property and its average time on the market (e. g. see Trippi, 1977; Miller, 1978; Merlo and Ortalo-Magné, 2004; de Wit and van der Klaauw, 2013).

[4] Since the seminal work of Burdett and Judd (1983) and Burdett and Mortensen (1998) search theory has been used to rationalize the existence of frictional price dispersion in several markets (most notably, in labor markets).

geneity of the economy generated by search and matching frictions, leads to substantial increases in the average housing price.

Our amplification mechanism critically relies on the inelasticity of the housing supply. For instance, in our benchmark economy, a 10 percent decrease in the down payment produces a 9.96 percent rise in the average housing price, leaving the homeownership rate almost unaltered. By contrast, in the opposite extreme case where the housing supply is infinitely elastic, the average price does not change but the homeownership rate rises from 69.42 to 76.21 percent. We also find that house price dispersion and average time on the market fall when credit is eased. The effect on the wealth distribution is complex due to the interaction of changes in liquidity and house prices. A rise in labor income produces similar effects. In general, our exercises suggest that increases (reductions) in housing demand reduce (increase) price dispersion and increase (reduce) average housing prices and the degree of market liquidity.

The work by Eerola and Maattanen (2018) is closely related to ours. These authors embed a random search model of the housing market into an heterogeneous agents framework with a fixed supply of identical houses. The model is calibrated using data of the Helsinki metropolitan area. The quantitative model is then used to study the steady-state effects of changing credit conditions and the asset holdings of home buyers and sellers on the degree of liquidity and frictional house price dispersion. The authors' key finding is that tighter credit conditions reduce overall liquidity and generate higher price dispersion, as in our model. Nevertheless, due to the lack of tractability of their model, they cannot study how market liquidity and credit conditions affect the homeownership rate and the wealth distribution. It is worth emphasizing that both their paper and ours imply that frictional price dispersion is higher in less liquid markets characteristics of a recession, which is intuitive, since competition among buyers is weaker in such markets.

Our framework is highly tractable because, as in the directed search models of Shi (2009) and Menzio and Shi (2010), the agents' value and policy functions do not depend the distribution of households across individual states. Instead, they depend on a finite dimensional variable which summarizes all the relevant information regarding the terms trade in the housing market. This is in contrast to random search models; e.g. see Molico (2006). In Shi (2009) and Menzio and Shi (2010) "block recursivity" arises from the combination of directed search and free entry of risk-neutral firms with constant returns in vacancy creation. In our framework, where the housing supply is fixed, it arises because we assume that home buyers and sellers do not trade directly with each other. Instead, trades are intermediated by agents with linear transferable utility who freely enter the market.[5]

An important contribution of this paper is theoretical. The simple recursive structure of our model allows us to prove existence of the households' value functions and to derive some of their properties. In particular, the value functions are shown to be differentiable along the optimal policies. This suffices to obtain the Euler equations. These results are not trivial since, due to the lack of concavity of the household's problem, the theorems of Mirman-Zilcha and Benveniste-Scheinkman do not apply to our setting. In recent work, Menzio et al. (2013) circumvent the technical difficulties arising from the non concavity by introducing lotteries. This makes the model tractable, but obviously not equivalent to the original problem, since the optimal policy functions differ. In this paper we do not need to introduce lotteries but work directly within the non concave framework. We also establish a link between the concavity of the value functions and the monotonicity of the optimal consumption policies.[6] Our computation strategy is based on these results, and so is the equilibrium characterization. To the best of our knowledge, our theoretical results are novel to the literature, and provide a new benchmark for solving similar block-recursive models with a non-degenerate asset distribution without the need of introducing lotteries.

Our other contribution is computational. To compute the model we modify the Endogenous Grid Method in Fella (2014) to include not only a non-convex choice (i.e., participate in the market or not) but also a continuous choice among submarkets that differ both in price and average time to buy. This method is much more efficient and accurate than standard Value Function Iteration, which is the methodology typically used in the recent literature that introduces search frictions into heterogeneous agents models. We comment on some related contributions in this literature below.

The rest of the paper is organized as follows. In Section 2 we describe our partial equilibrium environment and the problems solved by agents, and define a stationary equilibrium. Section 3 describes the block-recursive structure of our framework and presents the main theoretical results. Section 4 describes our calibration procedure and presents our main quantitative results. Section 5 concludes. Proofs and computational details are relegated to the Appendix.

[5] This is to avoid dealing with a setting with two-sided heterogeneity and two-sided risk aversion, which would be highly involved. Hedlund (2016a) develops a related model with construction where a different intermediation technology gives rise to block-recursivity. Yet his focus is different, and his approach is fully computational.

[6] To derive these results we adapt the approach recently introduced in Rincón-Zapatero (2019). This approach does not directly apply to the Bellman equations of our model (due to their particular structure) so additional work is required.

2 The model economy

In this section we present our model economy and define a stationary equilibrium.

2.1 Preferences and endowments

Consider a location populated by a continuum of households who live forever. Time is discrete. Households derive utility from the consumption of a divisible good and the service flow provided by an indivisible durable good which we refer to as *housing*. Their lifetime utility is $\sum_{t=0}^{\infty} E_0 \beta^t u(c_t, h_t)$, where $c_t, h_t \in \mathbf{R}_+$ are the amounts of the divisible good and housing services consumed each period, respectively, and β is the discount factor. The per-period felicity function u is strictly increasing, strictly concave and $\mathcal{C}^2$, with $u_{ch} \geq 0$ and $\lim_{h\to 0} u(c,h) = -\infty$. Households have a fixed endowment w of the divisible good every period, and can choose to either own or rent a housing unit. For simplicity, we assume that they value the services of a single unit and can own at most one unit each period. There is a fixed stock of owner-occupied housing which consists of H symmetric units which do not depreciate. We think of the housing market in our model as a secondary market.

Each period homeowners face idiosyncratic preference shocks which affect their consumption of housing services. Specifically, they can be in two idiosyncratic states, $\mu \in \{0, 1\}$. Owners in state 1 consume $\hbar > 0$ units of housing services, whereas owners in state 0 consume none. In words, owners in state 1 are matched with their home, and owners in state 0 are mismatched. The state μ follows a Markov process with transition probabilities $P(\mu' = 1|\mu = 1) = 1 - \pi_\mu \in (0, 1)$ and $P(\mu' = 0|\mu = 0) = 1$. So $\mu = 0$ is an absorbing state; owners can only transit out of this state by selling their home and moving to a new unit. By contrast, renters consume an exogenous amount h_r of housing services, where $0 < h_r \leq \hbar$. We thus allow for a taste for ownership.

Households face also idiosyncratic moving shocks (which are realized jointly with the owners' preference shocks). Owners and renters are hit by these shocks with different time-invariant probabilities, denoted by $\pi_{\xi_0}, \pi_{\xi_r} \in (0, 1)$, respectively.[7] All idiosyncratic shocks are independent across households. Households hit by a moving shock migrate to a *symmetric* location in the rest of the

[7] This is for calibration purposes since, in the data, renters move more often than owners; e.g. see Head et al. (2014).

world at no cost, and are instantaneously replaced by an equal mass of immigrants. The details on these entry flows are specified below. We normalize the constant measure of households in the location to one.

Our benchmark model abstracts away from idiosyncratic labor income risk and aggregate risk. We simply introduce the minimum amount of idiosyncratic risk needed to generate turnover in the housing market.

2.2 Market arrangements and real estate intermediation

Financial market arrangements are as in Díaz and Luengo-Prado (2010). Households can save by means of a risk-free asset with price $1/R \in \mathbf{R}_+$ (in units of the non-housing good). Their home purchases can be partially financed with a non-defaultable mortgage loan. Specifically, a household can borrow up to a fraction $(1-\delta)$ of the home's market value, so it must save in order to meet the corresponding down payment. The mortgage is a loan in perpetuity with no costs associated if there is early repayment. Houses also serve as collateral for loans: homeowners can obtain a home equity loan for up to a fraction $(1-\delta)$ of the home's value (i.e., they can always remortgage). There are indirect taxes on real estate transactions. Home sellers pay taxes on the value of the house at the rate τ_s, whereas the buyers' tax rate is τ_b. For simplicity, we assume that tax revenues are thrown away. Also, there is no spread between borrowing and lending rates, and households who do not own residential assets cannot borrow.

Real estate transactions are intermediated by agents with linear transferable utility who are free to enter the market each period. These agents purchase homes from mismatched owners, and then look for potential buyers.[8] Intermediaries are also infinitely-lived with discount factor β and can hold at most one unit each period. We do not model the rental market explicitly, and assume that any household who wants to rent a home can do so at a fixed price r_h.

We now specify the timing of the model, and describe the market structure in detail. Each period is divided into three subperiods: *morning*, *afternoon*, and *night*.

[8]This is not what most real estate agents do in reality. Yet it is a useful modeling choice which generates a simple block-recursive structure. We assume that intermediaries have deep pockets (and do not require credit to finance their purchases) and do not pay taxes.

2.2.1 Morning

At the start of a period, there are two types of households in the economy depending on their tenure status: *owners* and *renters*. First, preference and moving shocks are realized. A Walrasian market then opens where the owners who have been hit by these shocks supply their homes inelastically.[9] Intermediaries can freely enter this market to purchase a unit at the market clearing price, $\overline{p}$. Once the market closes, the households hit by the moving shock migrate and are replaced by an equal measure of immigrants who do not own residential assets.

Note that home sellers do not face trading delays in our model; it is intermediaries who face the inventory risk, as we shall see. Yet this risk will be priced into $\overline{p}$. We introduce the Walrasian market because it highly simplifies the analysis, allowing us to focus on buyers' outcomes.[10]

2.2.2 Afternoon

During the afternoon, those households who sold their home in the morning and did not migrate, those who were renters in the previous period, and the newly arrived immigrants decide whether to rent a home in the current period or search for a home to buy. We refer to these households as *potential buyers*. Matched owners make no economic decisions in this subperiod, so we refer to them as *non-traders*.

A competitive search market operates in the afternoon where intermediaries put their vacant homes up for sale at cost $\kappa_s > 0$, and buyers search for a unit which suits their needs at a negligible cost.[11] Home buyers may borrow up to a fraction $(1-\delta)$ of the home's value in the Walrasian morning market; i.e., their borrowing limit is $(1-\delta)\,\overline{p}$. The implicit assumption (as in Kiyotaki and Moore, 1997) is that banks lend the amount they can recover in the Walrasian market if they seized the house. Potential buyers may choose not to participate in the afternoon market (e.g. if

[9]This is optimal for mismatched owners as $\lim_{h\to 0} u(c,h) = -\infty$. It is also optimal for owners who are hit by the moving (but not the preference shock) if they face a sufficiently high cost of leaving their home unsold.

[10]See Hedlund (2016a) for a more complex setting where both buyers and sellers face trading delays, and where a different intermediation technology á la Lagos and Rocheteau (2005) gives rise to "block-recursivity". Hedlund studies the interaction between the credit market and the cyclical behavior of housing market aggregates. His main finding is that the increasing illiquidity characteristic of a slowdown in housing markets tightens credit constraints for borrowers (who find it harder to sell their home), creating a vicious circle that increases foreclosure activity and further depresses that the housing market.

[11]This cost is introduced to rule out equilibria where some households participate in the frictional market even though they do not plan to trade there (because doing so is costless).

they have not accumulated enough assets to meet the corresponding down payment).

The competitive search process is as in Moen (1997). Buyers and intermediaries can participate in different submarkets where they meet bilaterally and at random, and where each trader experiences at most one bilateral match. The probabilities with which buyers and intermediaries meet each other in a given submarket depend on the associated buyer-seller ratio θ (or market tightness). Specifically, an intermediary meets a potential buyer with probability $m_s(\theta)$, and a buyer meets an intermediary with probability $m_b(\theta) = m_s(\theta)/\theta$.[12] As is standard, $m_s(\theta)$ is strictly increasing, strictly concave and $\mathcal{C}^2$, with $m_s(0) = 0$ and $\lim_{\theta\to\infty} m_s(\theta) = 1$. Also, $m_b(\theta)$ is strictly decreasing and $\mathcal{C}^2$, with $\lim_{\theta\to 0} m_b(\theta) = 1$ and $\lim_{\theta\to\infty} m_b(\theta) = 0$. Intuitively, the higher the buyer-seller ratio θ, the easier it is for intermediaries to contact buyers and the harder it is for buyers to locate a vacant home for sale (due to congestion externalities). As θ goes to infinity (zero) the probability that an intermediary meets a buyer goes to one (zero), and the probability that a buyer meets an intermediary goes to zero (one). The elasticity $\eta(\theta) \equiv \frac{m_s'(\theta)\theta}{m_s(\theta)} \in [0,1]$ is assumed non increasing, and $\hat{m}_s(m_b) \equiv m_s(m_b^{-1}(\cdot))$ is such that $\ln \hat{m}_s$ is concave.[13]

To model market participation, it is useful to introduce a fictitious submarket $\theta_0 \in \mathbf{R}_-$, and extend the functions m_b and m_s to $\Theta \equiv \mathbf{R}_+ \cup \{\theta_0\}$ by setting $m_b(\theta_0) = m_s(\theta_0) = 0$. Households who choose θ_0 (rather than $\theta \in \mathbf{R}_+$) do not participate in the afternoon market.

To describe the price determination process in the competitive search market, we adopt the *price-taking* approach in Jerez (2014). The idea is to think of houses traded in submarkets with different tightness levels $\theta \in \mathbf{R}_+$ as different commodities, which are characterized by different degrees of trading uncertainty. The prices of these differentiated commodities are described by a continuous function $p : \Theta \to \mathbf{R}_+$, with $p(\theta_0) = 0$. That is, $p(\theta)$ is the housing price in a submarket with tightness $\theta \in \mathbf{R}_+$. Buyers and intermediaries have rational expectations about the tightness level prevailing in active submarkets, and choose the submarkets they enter taking $p(\theta)$ as given. As shown in Jerez (2014), our price-taking equilibrium notion is equivalent to that of directed search. In particular, $p(\theta)$ is the inverse of the schedule $\theta(p)$ describing the agents' beliefs (about the tightness level in submarkets with different prices) in Moen (1997) and Acemoglu and Shimer (1999). We choose the price-taking formulation because it makes the connection with the standard

[12]Implicit is the assumption that the total number of bilateral trading meetings is determined by a matching function with constant returns to scale, and that the Law of Large Numbers holds.

[13]That is, $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is non decreasing. This assumption guarantees that the problem solved by potential buyers is concave and has a unique solution (see Sections A-C in the Appendix), and can be further relaxed (see Section D.1). See also Menzio and Shi (2010) where $\hat{m}_s$ is assumed concave (a slightly stronger assumption).

notion of recursive competitive equilibrium more direct and transparent. The crucial difference with the standard competitive equilibrium notion is that the frictional afternoon market does not clear in equilibrium.

2.2.3 Night

Households who bought a home in the afternoon are *owners* at night, just as the non-traders. The rest of the households are *renters*. At night households receive the endowment w, and decide their consumption of the divisible good and the amount of assets to be carried to next period.

2.3 Stationary equilibrium

Below we state the problems of the agents in each subperiod (starting at night and going backwards), the law of motion of the distribution of households across individual states, and that of the vacancy stock held by intermediaries. A stationary equilibrium is then defined.

2.3.1 Night

Since intermediaries are inactive at night, we only need to describe the problem solved by households. Let $A = [\underline{a}, \infty)$ be the set in which financial assets can take values, and $a \in A$ be the amount of financial assets held by a household at the start of the night. The afternoon value functions of potential buyers and non-traders are $W_b : A \to \mathbf{R}$ and $W_n : A \to \mathbf{R}$, respectively. The night value function of an *owner* is then given by

$$
\begin{aligned}
W_o(a) = \quad & \max_{c, a' \in \mathbf{R}} \Big\{ u\left(c, \hbar\right) + \beta \, \left(1 - \pi_{\xi_o}\right) \left(1 - \pi_{\mu}\right) \, W_n\left(a'\right) \\
& \qquad\qquad\qquad + \beta \left[1 - \left(1 - \pi_{\xi_o}\right) \left(1 - \pi_{\mu}\right)\right] \, W_b\left(a' + \left(1 - \tau_s\right) \overline{p}\right) \Big\} \\
\text{s.t.} \qquad & c + \tfrac{1}{R} \, a' \leq w + a, \\
& a' \geq -\left(1 - \delta\right) \overline{p}, \\
& c \geq 0,
\end{aligned}
\tag{2.1}
$$

where c and $\hbar$ are the amounts of the divisible good and housing services consumed, and a' is the amount of financial assets carried to the next period. Owners choose the values of c and a'

to maximize their expected lifetime utility subject to a standard intertemporal budget constraint, and also face a borrowing limit equal to $(1-\delta)\,\overline{p}$. (As described earlier, they can remortgage their home, in which case the price of reappraisal is the Walrasian market price.) With probability π_{ξ_o}, owners will be hit by a moving shock in the next morning. If not, there is still a probability π_μ that they will become mismatched. An owner who is not hit by any of these shocks will be a non-trader in the next afternoon, with continuation value $W_n(a')$. An owner who is hit will sell her home at price $\overline{p}$ in the next morning and pay the corresponding indirect taxes. Note that the assumption that agents hit by the moving shock migrate to a symmetric location at no cost implies that the owner's continuation value is the same regardless of the kind of shock that hits her. Owners hit by the preference (but not by the moving) shock will be potential buyers in their current location, whereas owners hit by the moving shock will be potential buyers elsewhere. In both cases, their continuation value is $W_b(a' + (1-\tau_s)\,\overline{p})$. Denote the owners' optimal decision rules by $g_o^c(a)$ and $g_o^a(a)$.

The night value function of a *renter* is defined in a similar way:

$$\begin{aligned}
W_r(a) = \quad & \max_{c,a' \in \mathbf{R}} \left\{ u\,(c, h_r) + \beta\, W_b\,(a') \right\} \\
\text{s.t.} \quad & c + \tfrac{1}{R}\, a' \leq w - r_h + a, \\
& a' \geq 0, \\
& c \geq 0,
\end{aligned} \tag{2.2}$$

and $g_r^c(a)$ and $g_r^a(a)$ denote the optimal decision rules. The main difference is that these households pay the rent, r_h, and consume h_r units of housing services, and are not allowed to borrow. Also, their continuation value is not affected by the moving shocks they face. Whether or not they migrate, they will be potential buyers in the next afternoon (either in the current location or in a symmetric location elsewhere).

2.3.2 Afternoon

Let a be the household's financial assets at noon. *Non-traders* are inactive during the afternoon, so their value function is given by

$$W_n(a) = W_o(a), \tag{2.3}$$

Consider now the problem faced by *potential buyers.* These agents choose the submarkets they join taking as given the price schedule, $p(\theta)$, and the maximum loan they can obtain, $(1-\delta)\,\overline{p}$. Their value function is given by

$$\begin{aligned} W_b(a) = \quad & \max_{\theta\in\Theta} \Big\{ m_b\,(\theta)\; W_o\,(a-(1+\tau_b)\,p(\theta)) + (1-m_b\,(\theta))\; W_r\,(a) \Big\} \\ \text{s. t.} \qquad & a-(1+\tau_b)\,p(\theta) \geq -(1-\delta)\,\overline{p} \text{ if } \theta \in \mathbf{R}_+, \end{aligned} \tag{2.4}$$

and $g_b^\theta(a)$ denotes their optimal decision rule. The collateralized borrowing constraint in (2.4) ensures that households who join submarket $\theta \in \mathbf{R}_+$ have enough assets to pay for the down payment and the taxes associated to the transaction. With probability $m_b(\theta)$ these households buy a home and enter the night with financial assets $a-(1+\tau_b)\,p(\theta)$.[14] With complementary probability, they do not trade and carry their full assets into the night, when they will be renters (which is also what happens to potential buyers who choose not to participate in the afternoon market).

Likewise, *intermediaries* choose the submarkets they join in order to maximize their expected lifetime value given $p(\theta)$, so their expected value in the afternoon is

$$J = \max_{\theta\in\mathbf{R}_+} \Big\{ -\kappa_s + m_s\,(\theta)\; p(\theta) + (1-m_s\,(\theta))\;\beta\,J \Big\}. \tag{2.5}$$

Intermediaries who join submarket $\theta \in \mathbf{R}_+$ pay the cost κ_s and sell their unit with probability $m_s(\theta)$ at price $p(\theta)$, in which case they exit the location. With complementary probability, they do not trade and must wait until the next afternoon, when they will continue to search for a buyer. We denote the set of optimal solutions for problem (2.5) by Θ_J.[15]

2.3.3 Morning

Recall that owners hit by a shock sell their home at price $\overline{p}$ in the morning (whereas the rest of the households are inactive in this subperiod). By free entry, the expected profits of the intermediaries

[14] Households with a mortgage have negative assets at night and pay interests on that debt, as implied by the intertemporal budget constraint in problem (2.1).

[15] As is standard in these models, in equilibrium Θ_J includes a continuum of elements, intermediaries being indifferent between all $\theta \in \Theta_J$ (see Section 3).

who enter the Walrasian market are zero in equilibrium:

$$\overline{p} = J. \tag{2.6}$$

2.3.4 Stationary equilibrium definition

In order to define a stationary equilibrium, we also need to describe the evolution of the distribution of agents across individual states and over time. Let us first focus on households. The distribution of non-traders and potential buyers at noon is described by the Borel measures $\psi_n, \psi_b \in M_+(A)$, respectively. Similarly, $\psi_o, \psi_r \in M_+(A)$ represent the distribution of owners and renters at night. We use primes to denote the corresponding measures in the next period. Since the total mass of households is one,

$$\psi_n(A) + \psi_b(A) = \quad 1, \tag{2.7}$$

$$\psi_r(A) + \psi_o(A) = \quad 1. \tag{2.8}$$

We describe the asset distribution of immigrants by an exogenous probability measure $\zeta_i \in P(A) \subset M_+(A)$. Since net migration flows are zero, the inflow of immigrants is given by $\psi_i \in M_+(A)$ with

$$\frac{\psi_i}{\psi_i(A)} = \quad \zeta_i, \tag{2.9}$$

$$\psi_i(A) = \quad \pi_{\xi_r}\psi_r(A) + \pi_{\xi_o}\psi_o(A). \tag{2.10}$$

Let $\mathcal{A}$ denote the Borel σ-algebra on A. Define the transition function $Q_o : A \times \mathcal{A} \to [0, 1]$ which gives the probability that an owner holding $\tilde{a} \in A$ assets at night will carry an amount of assets in $X \in \mathcal{A}$ into the next morning. Likewise, Q_r denotes the corresponding transition function for renters.[16] The laws of motions from the night to the following afternoon are

$$\psi_n'(X) = \quad (1 - \pi_\mu)(1 - \pi_{\xi_o}) \int_{a \in A} Q_o(a, X) d\psi_o, \tag{2.11}$$

$$\psi_b'(X) = \quad (1 - \pi_{\xi_r}) \int_{a \in A} Q_r(a, X) d\psi_r + \pi_\mu (1 - \pi_{\xi_o}) \int_{a \in A} Q_o(a, X) d\psi_o + \psi_i(X), \tag{2.12}$$

[16] That is, $Q_j(a, X) = \psi_j(\{a \in A : g_j^a(a) \in X\})$ for $j \in \{o, r\}$.

for each $X \in \mathcal{A}$. Similarly, the laws of motion from the afternoon to the night are

$$\psi_o'(X) = \quad \psi_n(X) + \int_{a \in A} \Pi_o(a, X)\, d\psi_b, \tag{2.13}$$

$$\psi_r'(X) = \quad \int_{a \in A} \Pi_r(a, X)\, d\psi_b, \tag{2.14}$$

where the transition functions $\Pi_o : A \times \mathcal{A} \to [0,1]$ and $\Pi_r : A \times \mathcal{A} \to [0,1]$ give the probability that a potential buyer holding a assets at the start of the afternoon will be an owner or a renter with assets in X at night, respectively.[17]

A measure $b \in M_+(\mathbf{R}_+)$ describing the distribution of buyers across submarkets is easily constructed from ψ_b and g_b^θ:

$$b(\Xi) = \psi_b\left(\{a \in A : g_b^\theta(a) \in \Xi\}\right), \qquad \text{for all Borel } \Xi \subset \mathbf{R}_+. \tag{2.15}$$

That is, $b(\Xi)$ is the measure of buyers who participate in a submarket $\theta \in \Xi$. Similarly, we describe the distribution of intermediaries across submarkets by $s \in M_+(\mathbf{R}_+)$. The support of s contains elements $\theta \in \Theta_J$ which solve problem (2.5). The precise distribution s on Θ_J will be determined by rational expectations (see equation (2.20) below). The set of active submarkets (which attract both buyers and intermediaries) is given by the intersection of the supports of b and s.

It remains to describe the law of motion of the vacancy stock held by intermediaries. Let $\widetilde{V}$ be the stock at the start of a period, which is equal to the mass of intermediaries who did not sell their vacant units in the previous period. The mass of new intermediaries who enter the Walrasian morning market, ΔV, is equal to the number of units supplied in this market (by the owners who are hit by either a moving or a preference shock), since this market clears in equilibrium. That is,

$$\Delta V = [\pi_{\xi_o} + \pi_\mu\ (1 - \pi_{\xi_o})]\ \psi_o(A). \tag{2.16}$$

[17] These probabilities are related to the probability that the buyer purchases a home in the afternoon, which depends on the submarket θ she joins. A successful trade implies, not only a change in tenure status, but also a change in the financial assets (which again depends on θ). Specifically,

$$\Pi_o(a, X) = \begin{cases} m_b\left(g_b^\theta(a)\right), & \text{if } a - (1+\tau_b)\, p\left(g_b^\theta(a)\right) \in X, \\ 0, & \text{otherwise,} \end{cases}$$

$$\Pi_r(a, X) = \begin{cases} 1 - m_b\left(g_b^\theta(a)\right), & \text{if } a \in X, \\ 0, & \text{otherwise.} \end{cases}$$

Hence, the vacancy stock held by intermediaries at noon is

$$V = \widetilde{V} + \Delta V. \tag{2.17}$$

These are the units which are up for sale in the different afternoon submarkets:

$$V = \int_{\theta \in \mathbf{R}_+} ds = s(\Theta_J). \tag{2.18}$$

Finally, those intermediaries who do not trade in the afternoon carry their inventories into the next period:

$$\widetilde{V}' = \int_{\theta \in \mathbf{R}_+} [1 - m_s(\theta)]\, ds. \tag{2.19}$$

We are now ready to define a stationary equilibrium.

***Definition* 1.** *A recursive stationary equilibrium for this economy, given the interest rate $R-1$, the rental housing price r_h and the probability distribution of the immigrants' asset holdings ζ_i, is a list of value functions and optimal decision rules for the households $\left\{W_o, W_r, W_n, W_b, g_o^c, g_o^a, g_r^c, g_r^a, g_b^\theta\right\}$, a value J and a set Θ^J of optimal decisions for intermediaries in the afternoon, prices $(\overline{p}, p(\cdot))$, Borel measures $\{\psi_o, \psi_r, \psi_n, \psi_b, \psi_i, b, s\}$, and positive real numbers $(V, \Delta V)$ such that:*

1. *Households' optimality: $\left\{W_o, W_r, W_n, W_b, g_o^c, g_o^a, g_r^c, g_r^a, g_b^\theta\right\}$ solve the households' problems in* (2.1)–(2.4) *given $(\overline{p}, p(\cdot))$.*

2. *Intermediaries' optimality and zero profits: J and Θ_J solve the intermediary's problem in* (2.5) *given $p(\cdot)$, and the zero profit condition in* (2.6) *holds.*

3. *All agents have rational beliefs about the tightness levels prevailing in active submarkets during the afternoon:*

$$\int_{\theta \in \Xi} db = \int_{\theta \in \Xi} \theta\, ds, \qquad \textit{for all Borel } \Xi \subset \mathbf{R}_+, \tag{2.20}$$

where b is given by (2.15), *and $\operatorname{supp} s \subset \Theta_J$.*

4. *The Walrasian morning market clears: ΔV is given by* (2.16).

5. *The vacancy stock at the start each period is stationary: $\Delta V = \int_{\theta \in \mathbf{R}_+} m_s(\theta)\, ds$.*

6. *The total number of homes that are either owner-occupied or for sale in the afternoon is equal to the housing stock:* $V + \psi_n(A) = H$.

7. *The stationary probability measures* $\{\psi_o, \psi_r, \psi_n, \psi_b, \psi_i\}$ *satisfy equations* (2.7)–(2.10) *and the laws of motion in* (2.11)–(2.14).

The only equilibrium condition that is not self-explanatory is the rational expectations condition in equation (2.20). This condition ensures that the measures of buyers and intermediaries in each active submarket are consistent with the tightness levels that agents take as given when they make their optimal afternoon decisions. Intuitively, ds represents the density of intermediaries and db represents the density of buyers in the set of active submarkets. If the traders' conjectures about the buyer-seller ratio θ are correct then (roughly speaking) db should be equal to $\theta\, ds$.[18]

3 Block-recursivity and equilibrium characterization

In this section we show that the equilibrium is block-recursive and exploit this feature to derive some properties of the value and policy functions, and to characterize the equilibrium sorting pattern in the frictional market.

3.1 Equilibrium price schedule

In equilibrium, the price schedule $p(\cdot)$—an infinite dimensional object which is used to calculate the households' value and policy functions—is pinned down by the value of $\overline{p}$. This allows us to get around the curse of dimensionality to compute the equilibrium.

This result is easily derived. Combining (2.5) and (2.6) yields:

$$p(\theta) \leq \frac{\kappa_s + (1-\beta)\,\overline{p}}{m_s(\theta)} + \beta\,\overline{p}, \text{ for all } \theta \in \mathbf{R}_+, \text{ with strict equality if } \theta \in \Theta_J. \tag{3.1}$$

In active submarkets, (3.1) then holds with equality. In particular, prices are lower in submarkets where θ is higher (since m_s is strictly increasing). Intuitively, since intermediaries get the same

[18] Formally, (2.20) says that b is absolutely continuous with respect to s, with Radon-Nikodym derivative θ. The supports of b and s then coincide almost everywhere, and this common support gives the set of submarkets which are active in equilibrium. Condition (2.20) replaces the standard market clearing condition in frictionless models. See also Peters (1997), Eeckhout and Kircher (2010), and Jerez (2014).

expected payoff J in all active submarkets and the probability of completing a sale increases with θ, prices must be higher in those active submarkets where θ is lower. On the other hand, prices in inactive markets imply a (weakly) lower expected payoff for intermediaries.

In fact, there is no loss of generality in assuming that (3.1) holds with equality *for all* $\theta \in \mathbf{R}_+$, so prices are such that intermediaries get the same payoff in all submarkets, whether active or not. A standard feature of general equilibrium models with a continuum of commodities is that prices in inactive markets are indeterminate; e.g. see Gretsky et al. (1999). Selecting a function $p(\theta)$ which satisfies (3.1) with equality for all $\theta \in \mathbf{R}_+$ is equivalent to selecting the highest prices that support the equilibrium allocation in our model.[19] With this price selection rule, $p(\theta)$ is pinned down by $\overline{p}$. By the zero profit condition, $J = \overline{p}$, so $\overline{p}$ is the average return from a vacant home for sale in the frictional market. As shown in Figure 1, the selected price function is strictly convex and $\mathcal{C}^2$ (because m_s is strictly concave and $\mathcal{C}^2$). It is also bounded below by $p_{min} \equiv \kappa_s + \overline{p}$ (the sum of the Walrasian price and cost of posting a vacancy in the frictional market), which is the price intermediaries would charge if the probability of selling the house was one (in order to break even). Since trade is subject to rationing, no intermediary would trade at a price $p \leq p_{min}$.

The equilibrium of this economy is block recursive because the problems solved by potential buyers and intermediaries do not depend directly on the (infinite-dimensional) distribution of households over assets. They only depend on the Walrasian price (a scalar). Given $\overline{p}$, the agents know the equilibrium price schedule in the frictional market. This is all they need to know to make their optimal decisions. The households' asset distribution only affects the agents decisions through its effect on $\overline{p}$. This block-recursive structure arises because (1) search in the afternoon market is directed, and (2) intermediaries have linear transferable utility (i.e, they are risk-neutral and have deep pockets) and make zero profits in equilibrium.[20]

[19]As discussed in Jerez (2014), this price selection rule is equivalent to the restriction typically imposed on out-of-equilibrium beliefs in directed search models, known as the market utility property. See, for instance, Peters (1997), Shi (2009), Eeckhout and Kircher (2010), and Menzio and Shi (2010).

[20]Even without free entry, our arguments go through provided the mass of intermediaries who seek to buy homes in the Walrasian market is higher than the mass of home sellers in that market (so intermediaries make zero expected profits). Yet the argument would break down if there where excess supply in this market. In this case, home sellers would be rationed and intermediaries would make a positive expected profit, $J - \overline{p}$, which would depend on the sellers' asset distribution.

3.2 Properties of the value functions

In Appendix A we show that, given the selected price function, the dynamic programming problems (2.1)–(2.4) admit continuous solutions W_o, W_r, W_n and W_b which are unique in a suitable class of functions (under quite general conditions).[21] Also, W_o, W_r and W_n are strictly increasing and W_b is non-decreasing. Whereas these functions need not be concave and differentiable in general, in Appendix B we show that they are differentiable along the optimal paths. This is all we need to establish the validity of the Euler equations used in our computation. We also show that, if we restrict to the range of assets of the households who participate in the frictional market, W_o, W_r and W_o are strictly concave and W_b is concave provided the renters' consumption policy function, $g_r^c(a)$, is non-decreasing on this range. This is the case in our numerical analysis. In particular, due to the endogenous participation decision, W_b is not concave on A, but it is concave on the range of assets that correspond to participation (those $a \in A$ with $W_b(a) > W_r(a)$). We exploit this concavity to derive the sorting result and to characterize the participation threshold in the next section, and to compute the optimal policy function of a potential buyer, $g_b^\theta(a)$ (see Appendix D).

3.3 Sorting and participation in the frictional market

It is direct to show that buyers with different financial assets sort themselves out across submarkets with different prices and different trading probabilities. The optimal decision rule of the buyers who choose to participate is

$$\begin{aligned} g_b^\theta(a) \in \quad & \underset{\theta \in \mathbf{R}_+}{\arg\max} \left\{ W_r\left(a\right) + m_b\left(\theta\right) \, \left[W_o\left(a - \left(1+\tau_b\right) p(\theta)\right) - W_r\left(a\right)\right] \right\} \\ \text{s. t.} \quad & a - \left(1+\tau_b\right) p(\theta) \geq -(1-\delta)\, \overline{p}. \end{aligned} \tag{3.2}$$

The buyer's ex-post gains from trading at price p are given by

$$S(a,p) = W_o\left(a - \left(1+\tau_b\right) p\right) - W_r\left(a\right), \tag{3.3}$$

[21]The method of proof is classical, and is based on a contraction mapping theorem. See Theorem 1.

and realized with probability $m_b(\theta)$. Buyers choose the submarket $\theta \in \mathbf{R}_+$ where their expected gains are highest. Since W_o is differentiable, $g_b^\theta(a)$ satisfies the first-order condition

$$m_b'(\theta)\, S(a, p(\theta)) - m_b(\theta)\, W_o'\left(a - (1+\tau_b)p(\theta)\right)\, (1+\tau_b)\, p'(\theta) = \lambda(a)\, (1+\tau_b)\, p'(\theta), \tag{3.4}$$

where $\lambda(a)$ is the Lagrange multiplier of the borrowing constraint in (3.2). If the constraint is not binding, (3.4) simplifies to

$$\left(\frac{1}{1+\tau_b}\right)\left(\frac{1-\eta(\theta)}{\theta}\right)\left(\frac{S(a, p(\theta))}{W_o'\left(a - (1+\tau_b)\, p(\theta)\right)}\right) = -p'(\theta), \tag{3.5}$$

where $\eta(\theta)$ is the elasticity of $m_s(\theta)$. The left-hand side of (3.5) represents the buyer's marginal rate of substitution of θ for p. Equation (3.5) says that the buyer's optimal choice is characterized by a tangency between her indifference curve on the space (θ, p) and the equilibrium price function $p(\theta)$. The optimal choice of an unconstrained buyer then attains the highest indifference curve along the price function p, as depicted in Figure 1.

Since the schedule $p(\theta)$ corresponds to the intermediaries zero isoprofit line on the space (θ, p), the indifference curve of the buyer is tangent to this isoprofit line. This is the standard characterization of a directed search equilibrium in the absence of credit constraints (e. g. see Moen, 1997; Acemoglu and Shimer, 1999). The tangency condition (3.5) can then also be expressed as:

$$\left(\frac{1}{1+\tau_b}\right)\left(\frac{S(a, p(\theta))}{W_o'\left(a - (1+\tau)p(\theta)\right)}\right) = \frac{\eta(\theta)}{1-\eta(\theta)}\left(p(\theta) - \frac{\overline{p}}{R}\right). \tag{3.6}$$

The second term in the left-hand side of (3.5) represents the buyer's ex-post gains from trading in submarket θ in units of the no-housing good (as W_o' is the marginal utility of wealth of an owner at night). In turn, $(p(\theta) - \overline{p}/R)$ are the gains that accrue to intermediaries. In the absence of taxation ($\tau_b = 0$), (3.6) generalizes the well-known condition in Hosios (1990) for environments with transferable utility to our environment with one-sided risk aversion. This condition says that a fraction $\eta(\theta)$ of the bilateral trading surplus is appropriated by the buyer and a fraction $1 - \eta(\theta)$ goes to the intermediary.

If the borrowing constraint binds,

$$p\left(g_b^\theta(a)\right) = \frac{a + (1-\delta)\, \overline{p}}{(1+\tau_b)}. \tag{3.7}$$

Constrained buyers join the submarket where the housing price equals the maximum price they can afford to pay given the assets they have accumulated, the taxes involved in the transaction and the borrowing limit (see Figure 2).[22] These buyers start the night subperiod with a negative asset position equal to $-(1-\delta)\,\overline{p}$. As one would expect, for constrained buyers the shadow price of the borrowing constraint, $\lambda(a)$, decreases with a (see Lemma 1 in Appendix C). There are then three possible cases. Either all buyers are unconstrained, they are all constrained, or the constraint only binds below a threshold. The last case is the relevant one in our quantitative analysis, as we shall see.

For a given $\overline{p}$, the problem of a constrained buyer has a unique solution, characterized by (3.7). The same is true for unconstrained buyers provided $g_r^c(a)$ is non decreasing (and so W_o is concave) on the range of assets that correspond to participation. In this case, there is a single tangency point between the buyers' indifference curves and the price schedule $p(\theta)$.[23] This means that all buyers with the same level of financial assets join the same submarket in equilibrium.

***Proposition* 1.** *A solution to the problem of a potential buyer exists. Moreover, if $g_r^c(a)$ is non decreasing on the range of a for which $\theta_0 \notin g_b^\theta(a)$ then $g_b^\theta(a)$ is single-valued on this range.*

We now turn to the equilibrium sorting pattern. For constrained buyers, (3.7) implies that $p(g_b^\theta(a))$ increases and $g_b^\theta(a)$ decreases with a. Under the conditions of Proposition 1, this is also the case for unconstrained buyers provided the gains from trading at a given price p, $S(a,p)$, increase with a.[24] In this case, the buyers' indifference curves are steeper when a is higher (see Figure 3). In words, buyers with higher financial wealth are willing to accept a larger price increase in order to increase their trading probability (while remaining indifferent) relative to buyers with lower assets. This implies that in equilibrium wealthier buyers trade in more expensive submarkets where average buying times are shorter. This endogenous separation of different agent types across different submarkets is a typical property of directed search models with heterogeneity.

***Proposition* 2.** *(Sorting by financial assets). For constrained buyers, $g_b^\theta(a') < g_b^\theta(a)$ if $a < a'$. If*

[22] In this case, the sharing rule is distorted away from the Hosios rule (even if $\tau_b = 0$).

[23] Since $\eta(\theta)$ is non-increasing, one cannot conclude from (3.5) that the buyer's marginal rate of substitution increases along an indifference curve as θ rises (as depicted in Figure 1). One may circumvent this issue assuming that traders choose m_b rather than θ, since there is a one-to-one mapping between both variables. Under the assumption in Proposition 1, the indifference curves of buyers and the isoprofit lines of intermediaries in the space (m_b, p) have a strictly convex shape, so they are tangent at most one point.

[24] Again, this is the case in our computations. It is direct to check that a sufficient condition for this requirement is that the purchase of a home always implies a lower night consumption level: $g_o^c\,(a-(1+\tau_b)p_{min}) < g_r^c(a)$.

$g_r^c(a)$ is non decreasing the range of a for which $\theta_0 \notin g_b^\theta(a)$ and $S(a,p)$ increases with a for each $p \geq p_{min}$ then $g_b^\theta(a)$ is strictly decreasing on this range (whether or not buyers are constrained).

Regarding the participation decision, under the conditions in Proposition 2, there is a threshold $a_{part} \in A$ such that potential buyers with assets $a > a_{part}$ strictly prefer to participate, those with assets a_{part} are indifferent between participating or not, and the rest do not participate. Thus $W_b(a) > W_r(a)$ for all $a > a_{part}$, and $W_b(a) = W_r(a)$ for $a \leq a_{part}$. The optimal price paid by buyers with assets a_{part} (when they participate) is the lowest housing price in the frictional market.

Proposition 3. *(Participation) Suppose that $W_b(a) > W_r(a)$ for some a. Under the conditions in Proposition 2, there is a threshold $a_{part} \in A$ such that $g_b^\theta(a) \in \mathbf{R}_+$ if $a > a_{part}$, $g_b^\theta(a) = \theta_0$ if $a < a_{part}$, and $g_b^\theta(a_{part}) = \{\theta_0, \overline{\theta}\}$ where $\overline{\theta}$ is the tightness prevaling in the cheapest active submarket.*

4 Quantitative analysis

In this section we assess the quantitative properties of our theory. We start by briefly discussing the computation of the model economy and our calibration strategy. We then present the main statistics of the benchmark economy, as well as some comparative static results.

4.1 Computation

Before describing our computation method it is useful to compare our framework to related search models studying the interaction between wealth accumulation and frictional wage dispersion, such as Krusell et al. (2017), Chaumont and Shi (2018) or Eeckhout and Sepahsalari (2018). These models assume, as we do here, that only one side of the market (workers in their setting versus households in ours) is risk averse, whereas the other (firms seeking to fill job vacancies there versus intermediaries here) has linear transferable utility and is free to enter the economy. Our sorting result is the parallel of that of Chaumont and Shi (2018), whereby wealthier workers get higher wages and take longer to switch employment states.[25] There are some important differences though. First, whereas the housing stock is exogenous here, labor search models typically assume that firms

[25]See also Eeckhout and Sepahsalari (2018) and Herkenhoff (2018). Menzio et al. (2013) obtain a similar sorting result in the context of a monetary search model.

may create new vacancies each period at an exogenous fixed cost. The equivalent of this assumption in our setting would be to assume that intermediaries can build new homes at an exogenous cost. By free entry, the Walrasian morning price, $\overline{p}$, would be equal to this cost in equilibrium. We will return to this issue in Section 4.4, where we show that movements in the Walrasian price are crucial for changes in aggregate (e.g. financial) conditions to affect the distribution of frictional prices in our quantitative model. A second difference is that, in our setting, households use financial assets not only to smooth (non-housing) consumption but also to build equity to pay the down payment. This introduces a not trivial participation margin. Because of the endogenous participation decision, the household's problem in our model has an additional non convexity which is not present in the above labor models. A similar non convexity arises in the monetary search model of Menzio et al. 2013, where households accumulate real money holdings instead of financial assets with a positive return. Nevertheless, as shown in Section 3.2, in our setting the value functions turn out to have standard properties on the range of assets that corresponds to participation.[26]

The computation of the model is not straightforward. One possibility would be to discretize the choice of financial assets and use value function iteration to solve the household's problems (in both stages). This is the procedure used by Chaumont and Shi (2018), Eeckhout and Sepahsalari (2018) and Hedlund (2016b), for instance. The drawback of this approach is that, by discretizing the choice of financial assets, we would be limiting the number of submarkets that are active in equilibrium ex ante, which may bias our results on price dispersion, specially when we conduct policy exercises. We instead apply the results in Section 3.2 and we compute the policy functions by solving the Euler equations. Our computation uses the endogenous grid method (EGM hereafter). Specifically, we extend the procedure in Fella (2014) to our framework. Fella (2014) modifies the EGM to include a discrete control variable subject to exogenous non-convex adjustment costs, in addition to the standard continuous variable. His algorithm yields substantial gains in accuracy and computational time relative to standard techniques used to solve non concave problems. Here we have a discrete choice (participating or not in the frictional market) and two continuum choices (the choice of submarket and savings). Also, instead of exogenous adjustment costs, home buyers face endogenous trading delays, as the choice of submarket determines the probability of buying (which is 1 in Fella's case). Furthermore, an inspection of the problem of potential buyers described

[26]It should also be noted that transactions in labor and housing markets are fundamentally different, in particular, because households may participate in both sides of the housing market (both buying and selling houses). Also, interactions between workers and employers are inherently dynamic, while the housing transactions we describe are one-time transactions.

in (2.4) shows that the computation may be further complicated by the fact that $m_b(\theta)$ is convex. Consistently with our theoretical results, in our computed equilibria the buyer's problem is always concave and has a unique solution. In Appendix D we describe in detail the computation method we use to solve the household's problem as well as to find the stationary equilibrium.

4.2 Calibration

The model period is a month. We use the additively separable felicity function

$$u\left(c,h\right)=\frac{c^{1-\sigma}}{1-\sigma}+\log(h). \tag{4.1}$$

Recall that matched owners consume $\hbar$ housing services, and renters consume h_r. As in Chaumont and Shi (2018), the matching technology is given by

$$m_s(\theta)=\left(1+\theta^{-\gamma}\right)^{\frac{-1}{\gamma}}, \quad m_b(\theta)=m_s(\theta)/\theta, \tag{4.2}$$

with $\gamma>0$. We use this function instead of the standard urn-ball matching function because it has an extra degree of freedom in that γ governs the elasticity of $m_b(\theta)$ with respect to θ.[27] We will come back to this issue when we discuss our results. We take the set where financial assets take values to be bounded, $A=[\underline{a},\overline{a}]$. The lower bound is ensured by the borrowing constraint, whereas the upper bound is ensured by the fact that there is always a positive probability that households buy a home and thus run down their assets.

Table 1 shows the calibration of the benchmark economy. We interpret the endowment of the non-housing good as labor income and set $w=1000$. (Recall that there is no labor income risk.) We calibrate the preference and mobility shocks to match the following observations on household turnover. According to the National Association of Realtors (NAR), average tenure length for US homeowners is around ten years. As in Head et al. (2014), we have assumed that households move across locations and target the annual frequency of owners and renters moving across counties in the US, which is about 3.2 and 12 percent, respectively, according to the Census Bureau. The three targets combined are used to calibrate the probabilities of the three shocks, π_μ, π_{ξ_o}, and π_{ξ_r}.

[27] With the urn-ball matching process this elasticity is constant. Since our computation method requires a one-to-one relationship between θ and $m_b(\theta)$, we cannot use the standard (truncated) Cobb-Douglas matching function (which implies $m_b=1$ for any θ sufficiently low).

The risk aversion parameter is set equal to $\sigma = 2$, which is a standard value. Regarding the value of housing services, the ratio $h_r/\hbar$ is the key parameter which determines the homeownership rate in our model. We choose to target this rate for working age households in the SCF. The average homeownership rate for the 1989-2007 waves is 69.43 percent, and the implied value for $h_r/\hbar$ is 0.9992. We set the discount factor to match the median wealth to earnings ratio for working age renters observed in the data. The average of this statistic in the SCF is 0.3450.

Regarding the matching technology, a larger value of γ reduces the severity of search and matching frictions and, hence, median time to buy in the steady state. According to the NAR median time to buy is between 10 to 12 weeks. Thus, we set γ so that the median time to buy in the steady state is 11 weeks. We choose the owner occupied housing stock, H, to match the median housing wealth to earnings ratio for working age owners in the SCF, which is 2.7223.

The real return to the risk-free asset, R, is such that its implicit annual return is equal to 3.91 percent, as calibrated in Díaz and Luengo-Prado (2010). The American Housing Survey reports that the median housing cost for renters for the last 10 years is about 28 percent of income. This includes the cost of maintenance and utilities. Thus, we take r_h to be 25 percent of the monthly wage. We follow Díaz and Luengo-Prado (2008) and set the tax on home purchases, τ_b, equal to 2.5 percent, and the tax on sales, τ_s, to 6 percent. The cost of posting a vacancy, κ_s, is set to zero.

The parameter δ, which corresponds to the down payment, is set to 0.5 to match the median Loan To Value ratio observed in the Survey of Consumer Finances for working age households. The average of this statistic for the 1989–2007 waves is 41.04 percent. The reason for this calibration choice (instead of setting $\delta = 0.2$, which is a more typical number in the literature) is to have a meaningful tenure choice. Díaz and Luengo-Prado (2008) show that, in the absence of risk, all households make the same tenure choice regardless of their wealth–they all either rent or own, depending on the value of the user cost of owner occupied housing relative to the rental price of housing. Here households do face idiosyncratic risk but this risk is so small that all households participate in the afternoon market and end up owning a home for low values of δ. Will return to this issue in Section 4.4.

We have assumed that immigrants own no residential assets. Since we do not have a sensible way to calibrate the distribution of their financial assets, we assume that they all enter the location with zero assets.

4.3 The benchmark economy

We next discuss the main quantitative properties of our benchmark economy.

4.3.1 Household's policy functions

It is useful to examine the household's optimal decisions before discussing the main features of the equilibrium. Panel (a) of Figure 4 depicts the price paid by buyers with different financial assets, which is given by $g^p(a) \equiv p(g^\theta(a))$. Panel (b) depicts the associated trading probabilities. Potential buyers with financial assets below 1.4052 times annual earnings do not participate in the afternoon market. We refer to a potential buyer with this threshold level of assets, a_{part}, as the *marginal buyer*. This buyer faces a binding borrowing constraint and directs her search to a submarket where the price of a house is 2.6636 times her accumulated wealth. Upon completing a transaction, her mortgage amounts to 1.325 times her annual earnings, which is the collateralized borrowing limit, $(1-\delta)\,\overline{p}$. The probability that she buys a home is only 0.0783 though. The borrowing constraint binds for any buyer with financial wealth below 1.4641 times annual earnings. For constrained buyers, the probability of buying a home rises very rapidly with a. For instance, it reaches 0.3573 for the threshold level of assets above which the borrowing constraint no longer binds. Any buyer whose wealth is greater than 2.8795 times annual earnings buys a home upfront and does not get a mortgage loan. A buyer whose wealth is equal to this level completes a sale with probability 0.4455 and pays 2.78 times her annual earnings for the house.

Let us turn to the renters' night savings decision. As we can see in panel (c) of Figure 4, consumption falls with assets for poor renters (as they save to buy a home). As soon as they have accumulated enough assets to participate in the frictional market, consumption starts rising with assets. For instance, a renter with zero assets has to wait for 7.58 years to become a marginal buyer. In the event that she does not complete a purchase (the most likely event for a marginal buyer), she will increase her savings further and will direct her search to a submarket with a higher price in the following period, where the probability of trading is higher. This probability rises slowly with a for high wealth levels though, as shown in panel (b) of Figure 4. This and the fact that households discount the future imply that the savings policy function of a renter is smooth and standard for levels of financial assets above a_{part}. This function has a fixed point: renters whose financial assets are 1.4739 times their annual earnings consume their income and role over their wealth. Any renter with more assets depletes them as the probability of buying is not large enough to compensate for lower non-housing consumption. The renter at the fixed point (whose assets are 1.4739 times her annual earnings) directs her search to the submarket where the price is 2.7277 times her annual earnings and gets a mortgage below her borrowing limit. This also implies that, at a steady state, the support of the financial wealth distribution of renters is the interval $[0, 1.4739 \times 12\, w]$. The support is so compressed because the probability of buying, m_b, rises very slowly with a for high wealth levels. For instance, the largest point of the grid is equal to 20 times annual earnings. A renter so rich directs her search to a submarket where the price is 3 times her annual earnings and m_b is 0.6955. The elasticity of $m_b(\theta)$ is then key to determine the support of the financial wealth distribution of renters and, therefore, that of the price distribution in the frictional market.

Consider now the owners' saving decisions. We do not show their savings policy function because it is very smooth and concave, as owners are hit by a mistmatch or a moving shock only every 10 years on average. Since the model period is a month, their behavior is very similar to the case in which they face no idiosyncratic risk at all. The fixed point of their savings policy function corresponds to a value of a equal to -1.0441 times their annual earnings. Hence, in equilibrium all owners hold debt. Yet their net worth—the value of all assets minus liabilities—is positive, for the following reason. Take a home buyer who exhausts the borrowing limit and becomes an owner with a mortgage equal to $(1 - \delta)\, \overline{p}$. In equilibrium, the Walrasian price equals 2.65 times annual earnings, and recall that $\delta = 0.5$. Valuing her home at this price, the owner's net worth is equal to $\delta\, \overline{p}$, which amounts to 1.3250 times her earnings. Yet, if she sells her home, she will have to pay $\tau_s\, \overline{p}$ in taxes. Hence, her liquid wealth is $(\delta - \tau_s)\, \overline{p}$, which is 1.1660 times her earnings. An owner whose assets equal -1.0441 times her earnings (the fixed point of the policy function) holds a net worth, $a + \overline{p}$, equal to 1.6059 times her earnings, whereas her liquid wealth is 28.09 percent of her earnings.

To sum up, in this economy renters hold positive financial assets and all owners hold debt. Renters accumulate assets to finance a down payment. This is so because the probability that owners are hit by a shock is low and so is the interest rate of mortgages, $R - 1$, (given the discount factor β). In terms of net worth, owners' wealth is much more concentrated than that of renters (who only hold financial wealth).

4.3.2 Aggregate implications

Table 2 shows some selected targeted statistics of the benchmark economy. The homeownership rate is 69.43 percent, as in the data. The median of housing wealth to annual earnings ratio for working age owners in the data is 2.72. We take as the model counterpart for median housing wealth the median price paid by home buyers in the frictional market; the ratio of this median price to annual earnings also equals 2.72. The median Loan To Value Ratio in the model (and in the data) is calculated in the following way. We take the (working age) owners who hold negative financial assets (mortgages in the data). We then calculate the median of their financial assets and divide them by the median housing wealth. This statistic in the data is 41.04, whereas in the model it is 38.68 percent. The median wealth to earnings ratio for renters is 0.35 in the data, and 0.28 in the model, which is a bit low. But recall that we are assuming away any income risk that would result in precautionary savings. The median time to buy is 11.56 weeks in the frictional market, which is within the range of about 10–12 weeks reported by NAR.[28]

We next analyze the implications of the model for some key non targeted statistics. Take the rent-to-price ratio, a typical index used to measure the return to housing. According to Sommer and Sullivan (2018), in the data this ratio is between 8 and 15 percent. To calculate the equivalent statistic in the model we need to take a stand on which is the reasonable statistic for house prices. On the one hand, houses are sold at price $\overline{p}$ in the Walrasian market. On the other hand, we have the cross-sectional house price distribution in the frictional market. Note that, regardless of its purchasing price, the liquidation value of a house is $\overline{p}$. This is why we report the rent-to-price ratio as the annual rent divided by $\overline{p}$. This ratio is 9.43 percent in the steady state, which is within the range reported by Sommer and Sullivan (2018).

Consider now average time on the market (TOM), a typical index of housing market liquidity. Recall that the probability that a house is sold during the afternoon and thus average TOM varies across submarkets. Also, intermediaries who do not sell their units can always choose to join a different submarket in next period. All these intermediaries face the same (ex-ante) expected probability of selling in next period. Therefore, we can calculate the expected TOM associated to the decision to join a particular submarket. The median of the distribution of this variable is

[28]For each potential buyer with assets $a \geq a_{part}$, we calculate time to buy taking into account the fact that, if the buyer does not trade this period, she will have more assets in the next period and will then direct her search to a submarket with a higher trading probability.

10.26 weeks, which is within the range of 4 to 17 weeks reported by NAR. Another index of market liquidity is months supply, which is the ratio of vacancies over sales in a given month (e.g. see Hedlund, 2016b). In the data, the average of this ratio was 5.47 in 2017 according to NAR, whereas in our model it is 2.61. Finally, the model's vacancy rate matches that in the data although we have not used this statistic as a target.[29] In sum, the model does a good job in matching aggregate features of the housing market and household's portfolio in spite of its simplicity.

4.3.3 The distribution of prices and wealth

In this subsection we analyze the distributional implications of the model. As explained in Section 4.3.1, the equilibrium price distribution in the afternoon market is very compressed. The associated standard deviation is around 1 percent (see Table 2). This is mainly because agents face a tiny amount of idiosyncratic risk. In particular, there are no differences in labor earnings across households, which would propagate to the wealth distribution (in the absence of complete markets) and, from the latter, to the price distribution.

There are many empirical studies that suggest that frictional dispersion in housing prices is significant, as discussed in the Introduction, but we do not have many estimates of it. For instance, Lisi and Iacobini (2013) estimate an hedonic pricing model using Canadian data controling for residual price dispersion (i.e., not explained by hedonic housing attributes). They find that the mean of the prediction error falls from 16.50 to 14.15 percent when taking into account residual dispersion, whereas its standard deviation falls from 14.62 to 12.37 percent. We thus take the difference from 16.50 to 14.15 as an estimate of the mean prediction error due to frictional price dispersion; this number is 2.35 percent. Likewise, we take as an estimate of the standard deviation of the prediction error the difference 14.62 − 12.47, which is 2.25. The mean prediction error in our benchmark economy is 0.8489 percent (see Table 2), which accounts for 36.12 percent of the estimate by Lisi and Iacobini (2013). The coefficient of variation of prices is 0.9269 percent, which is about 42 percent of the dispersion not explained by observables in their paper.[30]

Another source of information on frictional price dispersion comes from Zillow, the online US

[29] The vacancy rate and months supply go hand in hand in the model (so it is impossible to get both statistics right simultaneously).

[30] Previous work by these authors applies the same methodology to a rich Italian dataset which includes information about both buyers and sellers (see Lisi and Iacobini (2012)). There they find that the standard error that can be attributed to frictional dispersion is about 4.56 percent for a sample of selected Italian cities.

real estate database. Zillow's methodology is applied to homogeneous sets of homes in a given geographical segment and combines information about physical attributes of the home and the land, among other things.[31] As reported in their website, at the national level "Zillow's accuracy has a median error rate of 5 percent. This means half of the home values in the segment are closer than the error percentage and half are farther off." Median errors are also reported at the county and MPA level, with substantial variation across segments for which enough data are available. For instance, for top metropolitan areas, median errors lie between 3 and 7 percent.[32] If a home's sale price is different from the Zestimate, we can attribute part of the difference to the presence of search and matching frictions in the housing market. To construct a model counterpart for this estimate, we calculate the average price in the afternoon market, which is 2 percent higher than the Walrasian price. Since the distribution of prices is very compressed, all house prices lie within a 5 percent range of the average price. Nevertheless, 59.25 percent lie out of the 1 percent range. The fact that we cannot match the magnitude of Zillow's error is also consistent with the widely held view that unobserved heterogeneity must explain part of the residual dispersion in house prices. Yet our comparable statistic is about the same order of magnitude, which leads us to conjecture that frictional dispersion may be a substantial part of the overall residual dispersion.

Recall that all immigrants enter the economy with zero assets. Because of sorting, poorer buyers trade with a lower probability in the frictional market and accumulate higher wealth to access a submarket where they are more likely to trade. This behavior and the turnover due to the preference and mobility shocks generate a non-degenerate distribution of financial assets. The Gini coefficient for renters' wealth is 0.5345, which is pretty high; Díaz and Luengo-Prado (2010) report a coefficient of 0.89 in the 1998 Survey of Consumer Finances. Valuing homes at price $\overline{p}$, the Gini of wealth for the total population is 0.2221. This is significant for a setting with such a tiny amount of uncertainty if we consider that the typical Gini coefficient for household wealth in the US is about 0.8 (see Díaz and Luengo-Prado, 2010).

It will be useful to compare our results with those of Eerola and Maattanen (2018) at this point. This should be done with caution since their environment is quite different. Specifically,

[31] Several recent studies on housing use data from Zillow and other similar websites. Piazzesi et al. (2015) document differential search patterns by buyers at the ZIP code level using data from California's website Trulia, and argue that these differential patterns can explain differences in the prices of houses with similar characteristics across ZIP codes. Guerrieri et al. (2013) study the link between within-city migration, different house price dynamics across neighborhoods, and gentrification.

[32] Median errors lie between 4 and 13 percent for 75 percent of the counties (if one restricts to counties for which at least 80% of the observations include sale prices). See https://www.zillow.com/zestimate/

their random search and bargaining model features two-sided risk aversion as home buyers and sellers trade directly with each other (there is no intermediation), and it is not block recursive because agents need to guess the entire wealth distribution to take decisions (which highly limits its computational tractability). Whereas matching probabilities are exogenous, not all matches between buyers and sellers lead to trade. Thus buying times and TOM depend on the fraction of successful matches.[33] Additionally, their model features idiosyncratic labor income risk. The coefficient of variation of prices in our stationary equilibrium is twice the level they report. Yet we do not know how much of this difference is due to the random search assumption and how much is due to two-sided risk aversion.

4.4 Comparative statics

We now discuss the steady state effects of changes in some parameters of interest. To understand better these results, it is instructive to compare our economy with an alternative one where the stock of owner-occupied housing is allowed to respond to market conditions. The alternative economy is constructed in the following way. We fix $\overline{p}$ at its stationary value in our model, and assume that intermediaries who enter the economy can build a house at cost $\overline{p}$ before the afternoon market opens. We refer to this as the *construction* economy, as opposed to our economy with a *fixed* housing stock. As we discussed at the beginning of Section 4.1, this alternative economy parallels directed search models of the labor market where firms create a vacancy by a paying an exogenous cost. This cost in turn determines the relationship between tightness levels and equilibrium prices (the equivalent of our price schedule, $p(\theta)$). The difference is that here the cost, $\overline{p}$, is endogenous.

4.4.1 Changes in credit conditions

We first inspect the role of credit. Consider a reduction in the down payment. Such a reduction eases financial constraints for two reasons. First, it reduces the amount of equity needed to participate in the frictional market and, second, it increases liquidity of residential assets allowing owners to smooth non-housing consumption against their housing collateral. Notice that since there is no capital in our setup the size of the economy is not affected.

[33] By contrast, in our model matching probabilities are endogenous because they depend on the tightness in the different active submarkets, but all matches lead to trade.

Table 3 describes the effects of a 10 percent reduction (increase) in δ, from 0.5 to 0.45 percent (0.55 percent, respectively). As shown in columns two and five, this produces about a 10 percent change of the opposite sign in the Walrasian price. Consider the case of a reduction in δ. It is instructive to look at Figure 5(a) first, which plots the equilibrium prices as a function of the assets of the buyers who participate in the frictional market. The reduction in δ affects this function in three ways. First, the function shifts upwards (from the benchmark blue curve marked as $\delta = 50\%$ to the red one marked as $\delta = 45\%$), as all participating buyers can afford to pay a higher price. Second, its domain widens as poorer potential buyers can now participate (a_{part} falls). Third, its shape changes, becoming more concave for high levels of assets. This is so because the probability of buying is an increasing concave function of the price and the households' marginal utility is decreasing. The combination of these two factors implies that, when δ falls, all buyers who participate are willing to pay higher prices, but the effect is stronger for poorer buyers. The three effects, combined with the endogenous change in the wealth distribution, determine the Walrasian price as well as the mean and variance of prices in the frictional market in the new steady state.

Figure 5(b) shows expected buying times as a function of a. This function also shifts (from the blue to the red curve) and becomes flatter for high levels of financial assets. Its domain, again, becomes wider, and there is an upward shift for high levels of assets so that buying times are slightly longer for wealthier buyers (with respect to the benchmark). The latter change follows because more buyers participate and they are all willing to pay higher prices, resulting in an increase in congestion in more expensive submarkets.

The overall effect of a 10 percent reduction in δ is a 10 increase in the Walrasian price, a rise in the median time to buy from 11.5670 to 12.0780 weeks, and a fall in price dispersion. In Table 3 we report three different measures of dispersion. The first is the coefficient of variation of prices, which falls from 0.92995 to 0.5065 percent. The second is the percentage of transaction prices out of the 1 percent range of the average price, which falls from 59.2480 to 42.9450 percent. The last one is the price range, which measures the ratio of the highest to the lowest price. Interestingly, this statistic rises from 1.0241 to 1.0741. The overall price dispersion falls because the renters' wealth distribution changes and also because of the aforementioned change in the shape of the price function. As we can see, the Gini coefficient of renters' wealth falls a bit from 0.5345 to 0.5275. The Gini coefficient for owners' wealth, however, does not change. While all owners hold more debt (the median Loan-To-Value ratio rises from 38.6840 to 43.8840 percent), their housing wealth is higher too (the median housing wealth to earnings ratio increases from 2.7049 to 2.9712), compensating their higher indebtedness. Finally, as median time to buy rises, median TOM and the vacancy rate fall, so the market becomes more liquid. Note that the homeownership rate rises slightly from 69.4280 to 69.4850. This is consistent with the fact that the participation rate rises moderately and buying times increase.

The opposite case where the credit conditions tighten (as δ rises from 50% to 55%) is symmetric in terms of its aggregate effects (see Table 3). The Walrasian price falls by 10% and the homeownership rate falls a slightly. The asymmetries show up in Figures 5(a) and 5(b) though. When δ rises, the curve which gives the equilibrium afternoon prices as a function of the buyers' assets shifts down and its support narrows (due to lower participation). The function also becomes steeper. This is so because buyers shift to submarkets with lower prices, higher congestion and longer time to buy (they move to the elastic part of $m^b(p)$). This results in an increase in price dispersion, as measured by the coefficient of variation, as the Gini coefficient of renters' wealth also rises. Median time to buy falls and the market becomes less liquid, with more vacancies and higher median TOM.

To have a sense of the importance of assuming a fixed housing stock, we also report the corresponding effects in the *construction economy*. The effect of a 10 percent decrease in δ in this economy is shown in column 3 of Table 3. The Walrasian price does not change (by assumption), but there is a large effect on distributional statistics. In particular, unlike in our benchmark, the homeownership rate is very sensitive to credit conditions. This rate rises from 69.4280 to 76.2090 percent, as housing becomes more affordable (the rent-to-price ratio increases), but neither time to buy nor TOM change. This is because the infinitely elastic supply of housing is effectively undoing the congestion effect brought about by the increase in housing demand. The Gini coefficient of renters' wealth falls significantly because buying times are shorter than in the benchmark economy. Consider now to the case where δ rises to 55%. In this case no renter wants to buy a house anymore, since they do not want to sacrifice non-housing consumption to build the high down payment. Hence, there are no home owners in the new steady state and the Gini coefficient of renters' wealth is zero, since renters do not face any risk. This exercise shows in a stark way that the congestion in the frictional market combined with an inelastic housing supply endogenously generates a significant amount of wealth heterogeneity.

To assess the quantitative significance of our results one should keep in mind that required down payments fell from about 20 to 1 percent during the period prior to the Great Recession, whereas the price-to-rent ratio, as reported by Favilukis et al. (2017), increased between 30 and 49 percent, depending on the price index used. Thus, a 95 percent fall in the down payment implied about a 40 percent increase in the price-to-rent ratio. In our benchmark, a 10 percent reduction in the down payment produces about a 9 percent change in the price-to-rent ratio (the inverse of the statistic reported in row seven of Table 3). These numbers suggest that the competitive search mechanism over-amplifies the effects of changes in financial conditions, but we should bear in mind that we are assuming a fixed housing stock and we are abstracting from idiosyncratic labor risk. It is useful to compare our results with those in Eerola and Maattanen (2018), who do allow for idiosyncratic income risk. These authors find that a 10 percent reduction in the down payment from 95 to 85 percent brings about a rise of 6 percent in the average housing price. It should be noted that, due to the complexity of computing their model, the homeownership rate is kept fixed in their quantitative analysis. These authors also find, as we do, that tightening credit conditions reduces liquidity (measured by average TOM) and increases price dispersion.

Our results stand in contrast with some of important results in the heterogeneous agents literature, which assumes a Walrasian (centralized) housing market where agents trade instantaneously at the equilibrium prices. For instance, Kiyotaki et al. (2011) and Sommer et al. (2013) find that changes in financial conditions have negligible effects on housing prices. The reason is that easing financial conditions affects only constrained agents, which represent a small fraction of the households in the economy. This is so in spite of the fact that Sommer et al. (2013) also assume a fixed housing stock. These authors find that the homeownership rate varies significantly, though, as in our construction economy. Favilukis et al. (2017) stress the importance of housing risk affecting all agents regardless of their wealth, along with a sufficiently high heterogeneity, for financial conditions to have a sizable aggregate effect on housing prices. They find that, in a model economy with these ingredients, a down payment reduction from 25 to 1 percent produces a 20 percent increase in the price-to-rent ratio. In Favilukis et al. (2017) housing risk is a byproduct of aggregate risk. Their main insight is that the combination of housing risk and high heterogeneity produces a significant number of "constrained" agents, which is the key for changes in financial conditions to affect prices.

Housing risk is indeed an essential part of our search environment: buyers purchase a home with certain probability and all home owners face a probability of realizing capital losses. Since search is directed, agents can affect the amount of risk they face through their savings and search decisions.

In particular, the severity of search and matching frictions affects the households saving decisions and thus the wealth distribution. Additionally, as we have seen, there is also a feedback effect from the wealth distribution to the frictions agents face in the housing market. The key point is that, in the language of the heterogeneous agents literature, all home buyers are effectively constrained in our model. When given one more euro, they all direct their search towards a submarket with a higher price, even if they do not face a binding credit constraint. This is why changes in financial conditions have such a large impact on the average housing price in our economy. This mechanism operates through the inherent heterogeneity of the economy which search and matching frictions generate. This is clear when we look at the exercises where no agent wants to buy a home; when all households are renters all of them hold the same level of wealth.

Our amplification mechanism critically relies on the inelasticity of the housing supply. Our results are consistent with Favara and Imbs (2015), who are able to identify a credit supply shock using US county and bank branches data for the period 1994-2005, and find that the response of prices to a credit shock depends on the response of the housing stock. There are many studies estimating housing supply elasticities. For instance, Green et al. (2005) or Gyourko et al. (2013) show that estimates of housing supply vary widely across cities depending on land abundance and particular regulation in each area. Moreover, they find that the cities which experienced a higher price boom were those with a low elasticity of housing supply and high income growth.

4.4.2 Other comparative static exercises

For the sake of completeness, we also report the steady state effects of a permanent change in the wage, the rental housing price, transaction taxes, and the interest rate in Table 4. The effect of a 5 percent increase in the wage, w, is reported in the third column. The first thing that stands out is the 9.42 percent increase in the Walrasian price. The effect on the rest of the variables is very similar to that of a reduction in the down payment. This may be surprising since, unlike a change in credit conditions, a change in the wage does affect the size of the economy. The reason the effect is so similar is that this is a partial equilibrium economy: we are ignoring the general equilibrium effects on production of the non-housing good and the interest rate. There is a difference, though, regarding the effect on the Median Loan To Value ratio, whose change after a rise in w is negligible and depends directly on the credit constraint. Additionally, a permanent change in the wage has, in relative terms, a larger effect on prices. All this suggests that changes in aggregate productivity, combined with an easing of credit conditions, should have a significant impact on housing prices.

It is worth mentioning that changes in transaction taxes—specially taxes on home purchases—have also a sizable effect on the Loan To Value Ratio. Finally, a reduction on the return to financial assets produces a small reduction in the Walrasian price as well as a corresponding increase in the homeownership rate. The effect on the price seems counterintuitive, but recall that agents do not face any income risk. Thus, if the interest rate falls, they change the composition of consumption: they consume more non-housing good. As a result, they save less and house prices fall.

5 Final comments

The key message that we want to convey is that changes in financial conditions and other shocks that affect housing demand, which typically do not have significant quantitative effects in standard heterogeneous agents models, are highly amplified in the presence of search and matching frictions.

We have developed a tractable framework to study how the interaction between search and matching and credit frictions and risk aversion jointly determines the price level and the degree of price dispersion in housing markets. We have assumed that owners are subject to idiosyncratic preference and moving shocks to generate housing turnover. Mismatched owners sell their home (without delays) in a Walrasian market intermediaries with linear transferable utility, who in turn sell the units to home buyers in a competitive search market. Households who own no residential assets do not face any exogenous risk: they have to move, but doing so is costless. The only uncertainty faced by non owners is that, if they choose to buy a home, they may be rationed in the search market.

Since search is directed, home buyers can ameliorate this uncertainty by directing their search towards more expensive submarkets where they are more likely to trade. This mechanism generates positive sorting in equilibrium, meaning that buyers with higher financial wealth pay higher prices and trade faster. Critically, in our theory, the average housing price depends crucially on the differential way in which the degree of market liquidity affects buyers with different financial wealth. The mechanism also creates wealth inequality: as potential buyers wait to find a trading opportunity, they save more to avoid queues. This is explains the connection between financial wealth distribution and the house price distribution. For instance, tightening credit increases wealth inequality and housing price dispersion and reduces the average housing price significantly. It also reduces housing market liquidity. The endogenous participation choice is key for our results, as we have seen. Our analysis also suggests that one should bare in mind that the effects of aggregate changes of this kind –and, in particular, the effects on liquidity and the distributional effects– critically depend on how whether or not the housing stock can adjust to those changes.[34]

We have made a number of strong simplifying assumptions. First of all, we have abstracted away from idiosyncratic labor income risk in order to study the amplification channel implied by competitive search. Second, in our setting mismatched owners can sell their property in a Walrasian housing market (where there is neither price dispersion nor trading delays). This eliminates part of the congestion that search and matching frictions create. This margin is important to understand why owners hold so little wealth in our model economy. Third, we have studied two extreme cases: an economy with a fixed housing stock and one in which the supply of housing is infinitely elastic. Both economies deliver strikingly different effects of changes in financial conditions and other shocks. The key issue that remains is the empirically relevant elasticity of supply.

We have also focused our attention on steady states. Studying the transitional dynamics of our model is not trivial, for the following reason. In our benchmark, the Walrasian price is endogenous. Out of the steady state this price depends not only on current economic conditions but also on expectations about future market liquidity. If potential buyers expect higher prices tomorrow they will start directing their search towards submarkets with higher prices today. Thus, expectations should play a key role out of the steady state. In the construction economy we have studied —which parallels directed search models of the labor market such as Menzio and Shi (2010) and Chaumont and Shi (2018)— this expectations effect is absent because, in equilibrium, the Walrasian price equals the fixed cost of building a house. We leave all these interesting extensions for future research.

[34]It is well-known that housing supply restrictions are key to understand housing markets (e.g. see Davis and Heathcote, 2005), and the increase in the overall housing price dispersion in the US, in particular (see Nieuwerburgh and Weill, 2010; Gyourko et al., 2013; Albouy and Zabek, 2016).

A Properties of the value functions

Let a denote the household's assets in a given subperiod (either night or afternoon). Recall that $A = [\underline{a}, \infty)$ and let $C(A)$ be the space of continuous functions $f : A \to \mathbf{R}$, and $E = C(A) \times C(A)$. Define the Bellman operator T on E by $T = (T_o, T_r)$, where

$$\begin{aligned} T_o(f_o, f_r)(a) = \quad & \max_{c,a'} \Big\{ u(c, \hbar) + \beta\, (1 - \pi_{\xi_o})(1 - \pi_\mu)\, f_o(a') \\ & \qquad + \beta\, [1 - (1 - \pi_{\xi_o})(1 - \pi_\mu)]\, T_b(f_o, f_r)(a' + (1 - \tau_s)\,\overline{p}) \Big\} \\ \text{s.t.} \quad & c + \tfrac{1}{R}\, a' \leq w + a, \\ & a' \geq -(1 - \delta)\,\overline{p}, \\ & c \geq 0, \end{aligned} \tag{A.1}$$

$$\begin{aligned} T_r(f_o, f_r)(a) = \quad & \max_{c,a'} \Big\{ u(c, h_r) + \beta\, T_b(f_o, f_r)(a') \Big\} \\ \text{s.t.} \quad & c + \tfrac{1}{R}\, a' \leq w - r_h + a, \\ & a' \geq 0, \\ & c \geq 0, \end{aligned} \tag{A.2}$$

and where $T_b(f_o, f_r)$ is defined by

$$T_b(f_o, f_r)(a) = \max \left\{ \max_{\theta \in D(a)} \Big\{ m_b(\theta)\, f_o(a - (1 + \tau_b)\, p(\theta)) + (1 - m_b(\theta))\, f_r(a) \Big\}, f_r(a) \right\}. \tag{A.3}$$

The feasible correspondence D of the inner maximization problem in (A.3) is defined by

$$D(a) = \{\theta \in \mathbf{R}_+ \,:\, a - (1 + \tau_b)p(\theta) + (1 - \delta)\overline{p} \geq 0\} \quad \text{for } a \in A. \tag{A.4}$$

If $D(a) = \emptyset$, we attach the value $-\infty$ to participation, and thus $T_b(f_o, f_r)(a) = f_r(a)$ in this case. Also, since

$$p(\theta) = \frac{\kappa_s + (1 - \beta)\overline{p}}{m_s(\theta)} + \beta\overline{p} \quad \text{for all } \theta \in \mathbf{R}_+, \tag{A.5}$$

$\lim_{\theta\to\infty} p(\theta) = p_{min}$. Since p is decreasing, $D(a) \neq \emptyset$ if and only if $a > (1 + \tau_b)p_{min} - (1 - \delta)\overline{p} \geq 0$. Since p is continuous in $\mathbf{R}_{++}$, D has closed sections. However, $D(a)$ is not compact. To circumvent this problem and be able to apply Bergé's Maximum Theorem, we assume that agents choose m_b rather than θ. Let

$$\hat{p}(m_b) = \frac{\kappa_s + (1 - \beta)\,\overline{p}}{\hat{m}_s(m_b)} + \beta\,\overline{p} \quad \text{for } m_b \in (0, 1), \tag{A.6}$$

and $\hat{p}(0) = p_{min}$. The function $\hat{p}$ is continuous in $[0, 1)$, since it is the composition of two continuous functions when $0 < m_b < 1$, and, for $m_b = 0$, $\lim_{m_b\to 0^+} \hat{p}(m_b) = \lim_{\theta\to\infty} p(\theta) = p_{min}$. Also, since $\hat{m}_s$ is strictly decreasing and $-\hat{m}_s{}'/\hat{m}_s$ is non decreasing, $\hat{p}$ is strictly increasing and strictly convex. Finally, $\lim_{m_b\to 1^-} \hat{p}(m_b) = \lim_{\theta\to 0^+} p(\theta) = \infty$. By choosing m_b as the new decision variable, the feasible correspondence D becomes $\overline{D}$, as defined by

$$\overline{D}(a) = \{m_b \in [0, 1) \,:\, a - (1 + \tau_b)\,\hat{p}(m_b) + (1 - \delta)\,\overline{p} \geq 0\}. \tag{A.7}$$

The sections of $\overline{D}$ are nonempty and compact for $a-(1+\tau_b)\,\hat{p}(m_b)+(1-\delta)\,\overline{p}>0$. In fact, when nonempty, $\overline{D}(a)$ is the bounded and closed interval $\left[0,\hat{p}^{-1}\left(\frac{a+(1-\delta)\,\overline{p}}{1+\tau_b}\right)\right]$. Problem (A.3) thus transforms into

$$T_b(f_o,f_r)(a)=\max\left\{\max_{m_b\in\overline{D}(a)}\left\{m_b\,f_o\left(a-(1+\tau_b)\,\hat{p}(m_b)\right)+(1-m_b)\;f_r\left(a\right)\right\},f_r(a)\right\}. \quad \text{(A.8)}$$

In Theorem 1 below, we assume that a positive level of consumption is always possible for both owners and renters. Since an initial wealth of $a=\underline{a}$ is admissible at the initial state, a positive consumption in the first period for owners and renters implies $w+\underline{a}+\frac{(1-\delta)\overline{p}}{R}>0$ and $w-r_h+\underline{a}>0$, respectively. In particular, the first inequality implies $w-\left(1-\frac{1}{R}\right)(1-\delta)\overline{p}>0$, which means that the owner can sustain a strictly positive level of consumption at the borrowing limit. Another consequence of the above inequalities is that $n_o=u\left(w+\underline{a}+\frac{(1-\delta)\,\overline{p}}{R},\hbar\right)>-\infty$ and $n_1=u(w-r_h+\underline{a},h_r)>-\infty$. This follows because the utility function $u(c,h)$ is finite if $c>0$, for $h\in\{\hbar,h_r\}$. On the other hand, utilities may be unbounded from above. Hence, we need to control for their rate of growth on the feasible correspondence, as well as for the size of β, to guarantee that the dynamic programming equations define a contraction operator. Consider the number sequence $\{a_0,a_1,\ldots,a_j,\ldots\}$, where

$$a_j=\left(\frac{Rw}{R-1}+\underline{a}\right)R^j-\frac{Rw}{R-1},\quad j=0,1,2,\ldots. \quad \text{(A.9)}$$

Note that $\underline{a}\leq a_j\leq a_{j+1}$, $a_j\to\infty$ as $j\to\infty$, and $a_0=\underline{a}$. Let

$$\begin{aligned}
u_j^o&=\max_{a\in[\underline{a},a_j]}\left|u\left(w+a+\frac{(1-\delta)\overline{p}}{R},\hbar\right)\right|,\\
u_j^r&=\max_{a\in[\underline{a},a_j]}\left|u\left(w-r_h+a,h_r\right)\right|,
\end{aligned}$$

and $u_j=\max\{u_j^o,u_j^r\}$. Note that both u_j^o and u_j^r are well defined because $n=\min\{n_o,n_r\}>-\infty$. Define

$$v_j:=\sum_{i=j}^{\infty}\beta^{i-j}u_j,\quad \text{for } j=0,1,2,\ldots. \quad \text{(A.10)}$$

The following theorem establishes the existence of a unique solution to the Bellman equation in a suitable class of functions. The result covers both the bounded and unbounded from below cases.

Theorem 1. *Suppose that $n>-\infty$ and that*

$$\lim_{j\to\infty}\frac{u_{j+1}}{u_j}:=\overline{u}<\frac{1}{\beta}. \quad \text{(A.11)}$$

Then, the dynamic programming equations (A.1), (A.2) *and* (A.3) *admit unique continuous solutions W_o, W_r and W_b, respectively, in the class of functions F defined by*

$$F=\left\{f\in C(A)\,:\,f(a)\geq\frac{n}{1-\beta},\ \textit{for all } a\in A,\ \max_{a\in[\underline{a},a_j]}f(a)\leq v_j,\ \textit{for all } j=0,1,\ldots\right\}. \quad \text{(A.12)}$$

Moreover, both W_o and W_r are strictly increasing and W_b is non decreasing.

Proof. Let $(f_o, f_r) \in E$. If $a \leq (1+\tau_b)\, p_{min} - (1-\delta)\overline{p}$ then the agent's optimal choice is θ_0, and so $T_b(f_o, f_r)(a) = f_r(a)$, which is continuous. When $a > (1+\tau_b)\, p_{min} - (1-\delta)\,\overline{p}$, the function $(m_b, a) \mapsto m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\ f_r\,(a)$ is continuous on $(a, m_b) \in A \times [0,1)$ and the correspondence $\overline{D}$ defined in (A.7) is nonempty valued, compact valued, and continuous. Hence, by the Theorem of the Maximum, the value function

$$\max_{m_b \in \overline{D}(a)} \left\{ m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\ f_r\,(a) \right\} \tag{A.13}$$

is continuous. Since $T_b(f_o, f_r)$ is defined as the maximum between this value function and f_r, it is also continuous. It follows that the functions defining the right hand side of $T_o(f_o, f_r)$ and $T_r(f_o, f_r)$ given in (A.1) and (A.2), respectively, are continuous. Moreover, the feasible correspondence is nonempty valued, continuous and compact valued in both cases. Hence, by the Theorem of the Maximum, both $T_o(f_o, f_r)$ and $T_r(f_o, f_r)$ are continuous. Let us see that $T_i(F \times F) \subseteq F$, for $i = o, r, b$. Let $(f_o, f_r) \in F \times F$. By the definition of T_b as the maximum of a convex combination of f_o and f_r, it is clear that $T_b(f_o, f_r) \geq \frac{n}{1-\beta}$. Also,

$$T_o(f_o, f_r)(a) \geq \max_{c,a'} u(c, \hbar) + \beta \frac{n}{1-\beta} \geq u\left(w + \underline{a} + \frac{(1-\delta)\overline{p}}{R}, \hbar\right) + \beta \frac{n}{1-\beta} \geq n + \beta \frac{n}{1-\beta} = \frac{n}{1-\beta}, \tag{A.14}$$

and

$$T_r(f_o, f_r)(a) \geq \max_{c,a'} u(c, \hbar) + \beta \frac{n}{1-\beta} \geq u(w - r_h + \underline{a}, h_r) + \beta \frac{n}{1-\beta} \geq n + \beta \frac{n}{1-\beta} = \frac{n}{1-\beta}. \tag{A.15}$$

On the other hand,

$$T_b(f_o, f_r)(a) \leq m_b\, f_o\,(a - (1+\tau_b)\,\hat{p}(m_b)) + (1-m_b)\ f_r\,(a) \leq m_b v_j + (1-m_b) v_j = v_j \tag{A.16}$$

and $T_b(f_o, f_r)(a) \leq f_r(a) \leq v_j$, for all $a \in [\underline{a}, a_j]$, for all $j = 0, 1, \ldots$. Hence, given that for any $a \in [\underline{a}, a_j]$, $\overline{D}(a) \subseteq [\underline{a}, a_{j+1}]$ by the definition of v_j, we have

$$T_o(f_o, f_r)(a) \leq u_j + \beta v_{j+1} = v_j, \quad \text{for all } a \in [\underline{a}, a_j]. \tag{A.17}$$

By a similar computation, $T_o(f_o, f_r)(a) \leq v_j$ for all $a \in [\underline{a}, a_j]$. It thus follows that $T_i(F \times F) \subseteq F$, for all $i = o, r, b$. Consider now $C(A)$ with the topology generated by the countable family of seminorms $\|f\|_j = \max_{a \in [\underline{a}, a_j]} |f(a)|$, for all $j = 0, 1, \ldots$. This family is separated ($\|f\|_j = 0$ for all j implies that f is the null function). Since the compact intervals $[\underline{a}, a_j]$ form an increasing family that covers A and they have nonempty interiors, the space $C(A)$ is complete with this topology (see Rincón-Zapatero and Rodríguez-Palmero, 2003). Consider the product space $E = F \times F$ with the seminorms $\|(f_o, f_r)\|_j = \max\{\|f_o\|_j, \|f_r\|_j\}$, for $j = 0, 1, \ldots$ and $(f_o, f_r) \in E$. It is clear that E is complete with this topology, and that the set E is closed. Consider the series $\sum_{j=0}^{\infty} c^{-j} u_j$, with $c > \overline{u}$, where $\overline{u}$ was defined in (A.11). By the ratio test and by (A.11)

$$\lim_{j \to \infty} \frac{c^{-(j+1)} u_{j+1}}{c^{-j} u_j} = \frac{\overline{u}}{c} < 1, \tag{A.18}$$

so the series converges. Moreover, since $\beta\, \overline{u} < 1$, it is possible to choose $c > \overline{u}$ with $\beta\, c < 1$. Following Theorem 4 in RZRP (2003), $T = (T_o, T_r)$ is a local contraction on $F \times F$, so T admits a unique fixed point in $F \times F$, that is, there are unique $W_o \in F$, $W_r \in F$ such that $T_o(W_o, W_r) = W_o$ and $T_r(W_o, W_r) = W_r$. Also, $T_b(W_o, W_r) = W_b$ is the buyer's value function.

To prove that W_o and W_r are strictly increasing, let $a_1 < a_2$. Then $\overline{D}(a_1) \subseteq \overline{D}(a_2)$, since $\hat{p}$, as the composition of two decreasing functions, is increasing. Let $(f_o, f_r) \in E \times E$, where both f_o and f_r are non decreasing. Then $m_b\, f_o\,(a - (1 + \tau_b)\,\hat{p}(m_b)) + (1 - m_b)\; f_r\,(a)$ is non decreasing in a, since $0 \leq m_b < 1$. Hence,

$$\begin{aligned}\max_{m_b \in \overline{D}(a_1)} & \left\{ m_b\, f_o\,(a_1 - (1 + \tau_b)\,\hat{p}(m_b)) + (1 - m_b)\; f_r\,(a_1) \right\} \\ & \leq \max_{m_b \in \overline{D}(a_1)} \left\{ m_b\, f_o\,(a_2 - (1 + \tau_b)\,\hat{p}(m_b)) + (1 - m_b)\; f_r\,(a_2) \right\} \\ & \leq \max_{m_b \in \overline{D}(a_2)} \left\{ m_b\, f_o\,(a_2 - (1 + \tau_b)\,\hat{p}(m_b)) + (1 - m_b)\; f_r\,(a_2) \right\}.\end{aligned}$$

It follows that $T_b(f_o, f_r)$ is continuous and, being the maximum of two non decreasing functions, it is also non decreasing. Plugging this result into the definitions of T_o and T_r, we get, by the same reasoning, that both $T_b(f_o, f_r)$ and $T_r(f_o, f_r)$ are non decreasing, since the feasible correspondence of both problems is increasing in a. Actually, both $T_b(f_o, f_r)$ and $T_r(f_o, f_r)$ are strictly increasing, since the utility function $u(w + a - a'/R, h)$ is increasing with respect to a, for $h \in \{h_r, \hbar\}$. Finally, the subset of F of non decreasing functions is closed in F, so the fixed points W_o, W_r and W_b are non decreasing. However, in the case of W_o and W_r, they are increasing by the previous argument, as they satisfy $T_o(W_o, W_r) = W_o$ and $T_b(W_o, W_r) = W_b$, respectively. $\square$

It is direct to show that the general theorem above applies, among others, to the utility functions used in the calibration of the model.

Corollary 2. *The conclusions of Theorem 1 hold under the same hypotheses, in the following cases.*

1. $u(c, h) = \dfrac{c^{1-\sigma}}{1 - \sigma} + v(h)$, *with* $\sigma > 1$.
2. $u(c, h) = \dfrac{c^{1-\sigma}}{1 - \sigma} + v(h)$, *with* $\sigma \leq 1$ *and* $R^{1-\sigma}\beta < 1$,

where $v(h) < v(\hbar)$. *Note that* $\sigma = 1$ *corresponds to* $u(c, h) = \log(c) + v(h)$.

Proof. We only need to show that (A.11) holds. Note that $u(\cdot, h)$ is increasing in cases 1 and 2. When $\sigma > 1$, u is negative and bounded. The sequence $\{u_j\}$, being increasing and bounded, then converges and $\overline{u} = 1 < \frac{1}{\beta}$. When $\sigma < 1$, u is positive but unbounded. In fact,

$$u_j^o = u\left(w + a_j + \frac{(1 - \delta)\overline{p}}{R}, \hbar\right). \tag{A.19}$$

Given the definition of a_j, it is direct to see that

$$\lim_{j \to \infty} \frac{u_{j+1}^o}{u_j^o} = \lim_{j \to \infty} \frac{\phi\left(w + a_{j+1} + \frac{(1-\delta)\overline{p}}{R}\right)^{1-\sigma} + v(\hbar)}{\phi\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)^{1-\sigma} + v(\hbar)} = R^{1-\sigma}. \tag{A.20}$$

In the logarithmic case, where $\sigma = 1$, u_j^o is bounded by $\left|\log\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)\right| + |v(\hbar)|$ for large enough j. The ratio

$$\frac{|\log\left(w + a_{j+1} + \frac{(1-\delta)\overline{p}}{R}\right)| + |v(\hbar)|}{|\log\left(w + a_j + \frac{(1-\delta)\overline{p}}{R}\right)| + |v(\hbar)} \tag{A.21}$$

tends to 1 as $j \to \infty$, so (A.11) is satisfied. A similar computation holds for u_j^r. $\square$

B Differentiability, Euler equations and concavity

In this section we prove the differentiability of the value functions along the optimal policies. This suffices to obtain the Euler equations; differentiability in the entire domain is not required. Other approaches to prove differentiability of the value function in a non-concave framework are due to Dechert and Nishimura (1983), Milgrom and Segal (2002), or Clausen and Strub (2016)), but do not apply to our setting (for the same reasons they do not apply to the model of Menzio et al. (2013)). Thanks to the results that we introduce in this section, we do not need to introduce lotteries but work directly within the non concave framework. We show that the Euler equations still hold as necessary conditions of optimality, so they can be used to compute the optimal policies. We are also able to establish a link between the concavity of the value functions and the monotonicity of the optimal consumption policies. Our results are based on the approach recently introduced in Rincón-Zapatero (2019). However, this approach does no apply directly to the Bellman equations satisfied by W_o, W_r and W_b, due to their particular structure, so we need to elaborate a bit more.

We introduce the concepts of Fréchet super– and subdifferentials of a function (F-superdifferential and F-subdifferential, henceforth) to simplify the presentation and the proofs that follow. For a continuous function $f : \Omega \subseteq \mathbf{R}^n \to \mathbf{R}$, where Ω is an open set, the vector $p \in \mathbf{R}^n$ belongs to the F-superdifferential of f at $x_0 \in \Omega$, $D^+f(x_0)$, if and only if there exists a continuous function $\varphi : \Omega \longrightarrow \mathbf{R}$ which is differentiable at x_0 with $D\varphi(x_0) = p$, $f(x_0) = \varphi(x_0)$ and $f - \varphi$ has a local maximum at x_0. Similarly, $p \in \mathbf{R}^n$ belongs to the F-subdifferential of f at $x_0 \in \Omega$, $D^-f(x_0)$, if and only if there exists a continuous function $\varphi : \Omega \longrightarrow \mathbf{R}$ which is differentiable at x_0 with $D\varphi(x_0) = p$, $f(x_0) = \varphi(x_0)$ and $f - \varphi$ has a local minimum at x_0. $D^+f(x_0)$ and $D^-f(x_0)$ are closed convex (and possible empty) subsets of $\mathbf{R}^n$. Yet, if f is differentiable at x_0, then both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty and $D^+f(x_0) = D^-f(x_0) = \{Df(x_0)\}$. Reciprocally, if for a function f, both $D^+f(x_0)$ and $D^-f(x_0)$ are nonempty, then f is differentiable at x_0 and $D^+f(x_0) = D^-f(x_0) = \{Df(x_0)\}$, where Df denotes the derivative of f. Given two continuous functions f_1 and f_2, two nonnegative numbers λ_1 and λ_2 and $p_i \in D^+f_i(x)$, for $i = 1, 2$, $\lambda_1 p_1 + \lambda_2 p_2 \in D^+(\lambda_1 f_1 + \lambda_2 f_2)(a)$. A similar proposition holds for D^-. Another property that we will use is that, whenever x_0 is a local maximum of f in Ω, $0 \in D^+f(x_0)$. Finally, $D^+f(x_0) \neq \emptyset$ if the function f is concave. See, for instance, Bardi and Capuzzo-Dolcetta (1997) for these and for other properties of the F-super– and subdifferentials of a function.

The next theorem characterizes the F-differentials of the value function $f(x) = \max_{y \in \Gamma(x)} F(x, y)$, where $F : X \times Y \to \mathbf{R}$ is continuous, with $X, Y \subseteq \mathbf{R}^n$, and where Γ is a correspondence from X to Y is nonempty, compact valued and continuous. The result is well known in the case in which the correspondence Γ is constant (i.e., when $\Gamma(x) = Y$ for all $x \in X$), but for the general case it is a generalization of the Benveniste-Scheinkman envelope argument. We will apply the theorem to show the validity of the Euler equations in our model, which is a non-trivial issue due to the lack of concavity.

Theorem 3. *Consider the problem described above, $f(x) = \max_{y\in\Gamma(x)} F(x,y)$. Let x_0 be an interior point of X and $y_0 \in \Gamma(x_0)$ satisfying: (i) $f(x_0) = F(x_0, y_0)$, and (ii) there is a ball $B(x_0, \varepsilon)$ in X with center x_0 and radius $\varepsilon > 0$, such that for all $x \in B(x_0, \varepsilon)$, $y_0 \in \Gamma(x)$. Then $D^-_x F(x_0, y_0) \subseteq D^- f(x_0)$ and $D^+ f(x_0) \subseteq D^+_x F(x_0, y_0)$, where $D^\pm_x F(x_0, y_0)$ denotes the F–upper/lower differential of the function $x \mapsto F(x, y_0)$.*

Proof. By Bergé's Theorem, f is continuous and the optimal policy correspondence is nonempty. Assumptions (i) and (ii) ensure that the function $x \mapsto f(x) - F(x, y_0)$ is well defined on the ball $B(x_0, \varepsilon)$ and attains a local minimum at x_0. If $D^-_x F(x_0, y_0)$ is empty, then there is nothing to prove. Suppose that it is nonempty. Let φ be continuous in $B(x_0, \varepsilon)$ and differentiable at x_0 such that $F(x, y_0) - \varphi(x)$ has a local minimum at x_0 and $F(x_0, y_0) = \varphi(x_0)$. Then $f(x) - \varphi(x) \geq F(x, y_0) - \varphi(x) \geq 0$ and $f(x_0) - \varphi(x_0) = F(x_0, y_0) - \varphi(x_0) = 0$ by (i). Thus x_0 is a local minimum of $f - \varphi$, and so $D\varphi(x_0) \in D^- f(x_0)$. Now, if $D^+ f(x_0) = \emptyset$ then $D^+ f(x_0) \subseteq D^+_x F(x_0, y_0)$, trivially. If $D^+ f(x_0) \neq \emptyset$, let φ be continuous in $B(x_0, \varepsilon)$ such that $D\varphi(x_0) \in D^+ f(x_0)$ and $f - \varphi$ has a local maximum at x_0, with $(f - \varphi)(x_0) = 0$. Then $F(x, y_0) - \varphi(x) \leq f(x) - \varphi(x) \leq 0 = F(x_0, y_0) - \varphi(x_0)$, for all $x \in B(x_0, \varepsilon)$. Hence, x_0 is a maximum of $x \mapsto F(x, y_0) - \varphi(x)$, and so $D\varphi(x_0) \in D^+_x F(x_0, y_0)$. $\square$

Remark 4. *Note that (ii) is satisfied when (x_0, y_0) is an interior point of the graph of Γ, although it may be fulfilled more generally, as we will show in our housing model. On the other hand, $D^-_x F(x_0, y_0) \neq \emptyset$ implies $D^-_x f(x_0) \neq \emptyset$. Hence, if f is concave then f is differentiable. This is the classical envelope theorem of dynamic programming.*

After this preliminary exposition, we turn to our specific problem, given by (A.1)–(A.3). In the results that follow, we will assume that there are selections of g^a_o, g^a_r and g^θ_b such that g^a_o and g^a_r are interior, and

$$0 \leq g^\theta(a) < p^{-1}\left(\frac{a}{1+\tau_b}\right), \tag{B.1}$$

for all $a \in A$. Hence, we do not assume uniqueness of the optimal policies.

Define $a_{min} = (1 + \tau_b)\, p_{min} - (1 - \delta)\, \overline{p}$. This is the threshold value of a above which $D(a)$, as defined in (A.4), is nonempty. Denote by $a_{part} > a_{min}$ the maximum $a > a_{min}$ such that $g^\theta(a) = \theta_0$ for $a_{min} < a \leq a_{part}$ (if it exists).

Our strategy for proving that the value functions are differentiable at the optimal policies consists on showing that both the F-subdifferential and the F-superdifferential are nonempty along the optimal policies. This is the content of the results that follow. As a byproduct, we prove that the Euler equations hold. We use this result in our computation (see Section D.3).

Lemma 5. *Let $a_0 > \underline{a}$. Then (i) $u_c(g^c_o(a_0), \hbar) \in D^- W_o(a_0)$, and (ii) $u_c(g^c_r(a_0), h_r) \in D^- W_r(a_0)$.*

Proof. We only prove (i), since the proof of (ii) is similar. W_o satisfies the Bellman equation (A.1). Since $g^a_o(a_0)$ is interior, given that the feasible correspondence is a closed interval, there is an open interval I centered at a_0, such that $g^a_o(a) \in (-(1-\delta)\overline{p}, R(w + a))$ for all $a \in I$. Thus (i) and (ii) in Theorem 3 hold. Moreover, taking $\alpha = (1 - \pi_{\xi_o})\,(1 - \pi_\mu)$, the function

$$F(a, g^a_o(a_0)) = u\left(w + a - g^a_o(a_0)/R, \hbar\right) + \beta\, \alpha\, W_o\left(g^a_o(a_0)\right) + \beta\,(1 - \alpha)\; W_b\left(g^a_o(a_0) + \overline{p}\right),$$

is differentiable with respect to a, with derivative $u_c(g_o^c(a_0), \hbar)$ at $a = a_0$, as the second and third summands in the definition of F are constant. Theorem 3 then implies $u_c(g_o^c(a_0), \hbar) \in D^- W_o(a_0)$. $\square$

To explore whether $D^- W_b$ is nonempty, we rewrite the problem of a potential buyer in an equivalent form. Let

$$W(a, m_b) = \begin{cases} W_r(a), & \text{if } a \leq a_{\min},\ m_b \in [0,1], \\ m_b \left(W_o(a - (1+\tau_b)\hat{p}(m_b)) - W_r(a)\right) + W_r(a), & \text{if } a > a_{\min},\ m_b \in \overline{D}(a), \end{cases} \tag{B.2}$$

where $\overline{D}(a)$ is defined in (A.7). Let $\widetilde{D}(a) = \{0\}$ for $a \leq a_{\min}$, and $\widetilde{D}(a) = \overline{D}(a)$ for $a > a_{\min}$. The correspondence $\widetilde{D}$ is nonempty, compact valued and continuous. Formally, we are identifying the choice θ_0 in the original problem with $m_b = 0$. Given this, it is clear that the original problem is equivalent to the new formulation: $\max W(a, m_b)$ subject to $m_b \in \widetilde{D}(a)$. Note that W is piecewise continuous and, when restricted to the graph of $\widetilde{D}$, it is continuous. For, if $(a_n, (m_b)_n)$ is a pair of sequences converging to $(a_{\min}, m_b)$ along the graph of $\widetilde{D}$, where $m_b \in [0,1]$, then for $a_n > a_{\min}$, $(m_b)_n = \widehat{p}^{-1}(a_n) \to \widehat{p}^{-1}(a_{\min}) = 0$, and for $a_n < a_{\min}$, $(m_b)_n = 0$. Hence,

$$W(a_n, (m_b)_n) \to 0(W_o(0) - W_r(a_{\min})) + W_r(a_{\min}) = W_r(a_{\min}) = W(a_{\min}, 0), \tag{B.3}$$

as $n \to \infty$. Since $m_b = 0$ is feasible for any a and $g_b^\theta(a) = 0$ in the region $a \leq a_{part}$ (if a_{part} exists), $W_b(a) = W_r(a)$ in this region.

Lemma 6. *Let $a_0 > \underline{a}$. Then $D^- W_b(a_0) = D^- W_r(a_0)$, for $a_0 \leq a_{part}$, and*

$$m_b\left(g_b^\theta(a_0)\right) p_o + \left(1 - m_b\left(g_b^\theta(a_0)\right)\right) p_r \in D^- W_b(a_0), \tag{B.4}$$

for $a_0 > a_{part}$, where $p_o = u_c\left(g_o^c(a_0 - (1+\tau_b)\, p(g_b^\theta(a_0))), \hbar\right)$ and $p_r = u_c(g_r^c(a_0), h_r)$.

Proof. For $\underline{a} < a < a_{part}$, $W_b(a) = W_r(a)$, so (i) is trivial. At $a = a_{part}$, $m_b = 0$ is the optimal choice (it is the only feasible choice, given our reformulation of the problem). Although not interior to the graph of $\widetilde{D}(a)$, this choice satisfies condition (ii) in Lemma 3, that is, $0 \in \widetilde{D}(a)$ in a neighborhood of a_{part} (for all a, actually). Hence, $D^- W_b(a_{part}) \neq \emptyset$. Let $a_0 > a_{part}$. Since g_b^θ is interior, the optimal $g^{m_b}(a_0)$ is interior. Thus the function of a

$$F(a, g^{m_b}(a_0)) = g^{m_b}(a_0) W_o(a - (1+\tau_b)\hat{p}(g^{m_b}(a_0))) + (1 - g^{m_b}(a_0)) W_r(a)) \tag{B.5}$$

is well defined in a suitable interval centered at a_0. Moreover, $D_a^- F(a_0, g^{m_b}(a_0)) \neq \emptyset$. To see this, take $p_o \in D^- W_o\left(a_0 - (1+\tau_b)\,\hat{p}(g^{m_b}(a_0))\right)$ and $p_r \in D^- W_r(a_0)$, which exist by Lemma 5. By one of the properties mentioned just above Theorem 3, $g^{m_b}(a_0)\, p_o + (1 - g^{m_b}(a_0))\, p_r \in D_a^- F(a_0, g^{m_b}(a_0))$, or, equivalently,

$$m_b\left(g_b^\theta(a_0)\right) p_o + \left(1 - m_b\left(g_b^\theta(a_0)\right)\right) p_r \in D_a^- F(a_0, g_b^\theta(a_0)), \tag{B.6}$$

with p_o and p_r as described in the statement of the lemma. Since $D_a^- F(a_0, g_b^\theta(a_0)) \subseteq D^- W_b(a_0)$ by Theorem 3, the result in the lemma holds. $\square$

The fact that the F-subdifferential of the value function is nonempty is not enough to get differentiability, since the value functions need not be concave. Below we follow the path initiated in Rincón-Zapatero (2019) to prove differentiability in the absence of concavity, which uses the optimality condition and the special structure of the Bellman equation. This will provide us with conditions for the nonemptiness of the F-superdifferential of the value functions at the optimal policies.

Lemma 7. *Let $a_0 > \underline{a}$. Then $u_c(g_r^c(a_0), h_r) \in R\beta\, D^+ W_b(g_r^a(a_0))$.*

Proof. Consider the Bellman equation (A.2) and the function of a' given by

$$F(a_0, a') := u(w - r_h + a_0 - a'/R, h_r) + \beta W_b(a'). \tag{B.7}$$

Since $g_r^a(a_0)$ is an interior optimum to the Bellman equation (A.2), $0 \in D^+_{a'} F(a_0, g_r^a(a_0))$. But, since u is of class C^1, $D^+_{a'} F = \{-u_c/R\} + \beta D^+ W_b$. Hence, $-u_c(g_r^c(a_0), h_r) \in R\beta D^+ W_b(g_r^a(a_0))$. □

Our next result shows that W_b is differentiable at the renter's optimal policy, and establishes the validity of the renter's Euler equation.

Proposition 8. *Let $a > \underline{a}$. Then*
(i) W_b is differentiable at $g_r^a(a)$;
(ii) the Euler equation

$$\begin{aligned} &- u_c(g_r^c(a), h_r) \\ &\quad + R\beta\Big[m_b\left(g_b^\theta(a')\right) u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) + \left(1 - m_b\left(g_b^\theta(a')\right)\right) u_c(g_r^c(a'), h_r)\Big] = 0 \end{aligned}$$

holds, where $a' = g_r^a(a)$.

Proof. By Lemma 6 and Lemma 7, both the F-super– and the F-subdifferential of W_b are nonempty at $g_r^a(a)$. Hence, W_b is differentiable at $g_r^a(a)$. The derivative is, on the one hand, the unique element in $D^- W_b(g_r^a(a))$, that is, $W_b'(g_r^a(a)) = \frac{1}{R\beta} u_c(g_r^c(a), h_r)$ and, on the other hand, the unique element in $D^+ W_b(g_r^a(a))$, that is

$$W_b'(g_r^a(a)) = u_c(g_r^c(a'), h_r), \tag{B.8}$$

for $g_r^a(a) \le a_{part}$, and

$$W_b'(g_r^a(a)) = m_b\left(g_b^\theta(a')\right)\Big[u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) - u_c(g_r^c(a'), h_r)\Big] + u_c(g_r^c(a'), h_r), \tag{B.9}$$

for $g_r^a(a) > a_{part}$, where $a' = g_r^a(a)$ in both (B.8) and (B.9). Since $m_b\left(g_b^\theta(a_{part})\right) = 0$, (B.9) encompasses (B.8). Equating $\frac{1}{R\beta} u_c(g_r^c(a), h_r)$ to (B.9), we obtain the renter's Euler equation. □

Proposition 9. *Let $a > a_{part}$. Then*
(i) W_o is differentiable at $g_o^a(a)$;
(ii) the Euler equation

$$\begin{aligned} -u_c(g_o^c(a), \hbar) + R\beta\alpha\, u_c(g_r^c(a'), \hbar) + R\beta(1-\alpha)\, m_b\left(g_b^\theta(a')\right) u_c\left(g_o^c(a' - (1+\tau_b)\, p(g_b^\theta(a'))), \hbar\right) \\ + R\beta(1-\alpha)\,(1 - m_b\left(g_b^\theta(a')\right)) u_c(g_r^c(a'), h_r) = 0, \end{aligned}$$

holds, where $a' = g_o^a(a) + \overline{p}$ *and* $\alpha = (1 - \pi_{\xi_o})\,(1 - \pi_\mu)$.

Proof. From (A.1), the function of a'

$$F(a, a') = u(w + a - a'/R, \hbar) + \beta\,\alpha\, W_o\,(a') + \beta(1 - \alpha)\, W_b\,(a' + \overline{p}) \tag{B.10}$$

satisfies $0 \in D^+_{a'} F(a, g_o^a(a))$. Since both u and W_b are differentiable,

$$-u_c(g_o^c(a), \hbar) + R\beta\,(1 - \alpha)\, W_b'(g_o^a(a) + \overline{p}) \in -R\beta\,\alpha D^+ W_o(g_o^a(a)),$$

so $D^+ W_o$ is nonempty at $g_o^a(a)$. This, combined with Lemma 5, implies that W_o is differentiable at $g_o^a(a_0)$. Also, its derivative at this point is given, on the one hand, by $u_c(g_o^c(a), \hbar)$, and, on the other hand, by $\frac{1}{R\beta\alpha} u_c(g_o^c(g_o^a(a)), \hbar) - \frac{(1-\alpha)}{\alpha} W_b'(g_o^a(a) + \overline{p})$. Equating both expressions, and replacing $W_b'(g_r^a(a))$ by its value in (B.9), we obtain the Euler equation in (ii). □

Now we study concavity. Concavity of the value functions is proved in intervals where the optimal consumption policy of the renters is nondecreasing.

Proposition 10. *W_b is concave in intervals I of the image of g_r^a if and only if g_r^c is nondecreasing in the preimage of this subset, $(g_r^a)^{-1}(I)$.*

Proof. Note that W_b is differentiable in I by Proposition 9. Also, if $a' \in I$, there is $a > \underline{a}$ such that $a' = g_r^a(a)$ and $W_b'(a') = u_c(g_r^c(\theta), h_r)/R$ by Lemma 5. Let $a_i' \in I$ and let $a_i > \underline{a}$ such that $a_i' = g_r^a(a_i)$, for $i = 1, 2$. Without loss of generality, suppose that $a_1' > a_2'$. By the Mean Value Theorem,

$$W_b(a_1') - W_b(a_2') = W_b'(\theta')(a_1' - a_2') = \frac{1}{R} u_c(g_r^c(\theta), h_r)(a_1' - a_2'), \tag{B.11}$$

where $a_2' < \theta' < a_1'$ and where $\theta' = g_r^a(\theta)$. Since g_r^a is nondecreasing, $a_2 < \theta < a_1$, and since g_r^c is non decreasing, $g_r^c(a_2) \leq g_r^c(\theta) \leq g_r^c(a_1)$. Now, $u(\cdot, h_r)$ is concave, so $u_c(g_r^c(\theta), h_r) \leq u_c(g_r^c(a_2), h_r) = RW_b'(a_2')$. Hence,

$$W_b(a_1') - W_b(a_2') \leq W_b'(a_2')(a_1' - a_2'), \tag{B.12}$$

and so W_b is concave in C. Obviously, the reasoning above is reversible. □

Proposition 11. *Let I be an interval of A such that $g_r^a(I)$ is an interval. Then W_r is strictly concave in I if and only if g_r^c is nondecreasing in I.*

Proof. Let $a_1, a_2 \in I$ and let $\lambda_1, \lambda_2 \in [0, 1]$. Since $g_r^a(I)$ is convex, $\lambda_1 a_1 + \lambda_2 a_2 \in I$, $\lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2) \in g_r^a(I)$. Also, $(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2))$ belongs to the graph of the buyer's feasible correspondence, since it is convex. Moreover, W_b is concave in $g_r^a(I)$ by Proposition 10. Then

$$\begin{aligned}
W_r(\lambda_1 a_1 + \lambda_2 a_2) &\leq u(\lambda_1 a_1 + \lambda_2 a_2, \lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2), h_r) + \beta W_b(\lambda_1 g_r^a(a_1) + \lambda_2 g_r^a(a_2)) \\
&\leq \lambda_1 u(a_1, g_r^a(a_1), h_r) + \lambda_2 u(a_2, g_r^a(a_2), h_r) + \beta\lambda_1 W_b(g_r^a(a_1)) + \beta\lambda_1 W_b(g_r^a(a_1)) \\
&= \lambda_1 W_r(a_1) + \lambda_2 W_r(a_2),
\end{aligned}$$

where we have used the fact that u is concave and W_b is concave in the image of g_r^a. Hence, W_r is concave in I. Strict concavity of W_r follows from strict concavity of u. $\square$

Proposition 12. *Let I be an interval of A such that both $g_o^a(I)$ and $g_r^a(I)$ are intervals and $\{\overline{p}\} + g_o^a(I) \subseteq g_r^a(I)$. Then W_o is strictly concave in I*

Proof. We use the fact that the restriction of the operator T_o to the set $\mathcal{F}$ is a contraction. This restricted operator is defined in the obvious way. First, fix the buyer's value function W_b which, given the hypotheses of the proposition and Proposition 10, is concave in $g_r^a(I)$. The restricted operator is then

$$T_o^b(f_o)(a) = \max_{c,a'} \left\{ U^b(c, \hbar) + \beta\alpha f_o(a') \right\}, \tag{B.13}$$

where $U^b(c, a') = u(c, \hbar) + \beta(1-\alpha)W_b(a' + \overline{p})$ is strictly concave, and $\alpha = (1 - \pi_{\xi_o})(1 - \pi_\mu)$. Hence, if f_o is concave, $T_o^b f_o$ is concave. By Stokey-Lucas-Prescott, the limit of the iterating sequence $(T_o^b)^n$ is concave and thus W_o is concave. Once this is proved, the dynamic programming equation in (B.13) implies that W_o is in fact strictly concave, since U_b strictly concave. $\square$

C Proofs of Propositions 1 to 3

The characterization results in Section 3.3 follow from the properties of the value functions established in Sections A and B. Potential buyers solve problem (A.8), or, equivalently, the problem described right after Lemma 5. Under the conditions of Theorem 1, an optimal solution to this problem exists, by the Theorem of the Maximum. Since the price function $\hat{p}$ in (A.6) is strictly increasing and strictly convex, the concavity result in Proposition 12 implies that, conditional on participating in the afternoon market, the optimal solution is unique under the assumptions in Proposition 1. Hence, by the Theorem of the Maximum, the associated policy function is continuous. This proves Proposition 1. Proposition 2 then follows from the differentiability W_o and the concavity result in Proposition 12.

Proof. Sice W_o is differentiable (Proposition 9), the optimal solution of buyers who find it optimal to participate in the afternoon market is characterized by the first-order condition:

$$\begin{aligned}&[W_o(a - (1+\tau)\hat{p}(m_b)) - W_r(a)] - m_b(1+\tau_b)\,\hat{p}'(m_b)\,W_o'(a - (1+\tau_b)\hat{p}(m_b))\\ &= \hat{\lambda}(a)(1+\tau_b)\,\hat{p}'(m_b),\end{aligned} \tag{C.1}$$

where $\hat{\lambda}(a)$ is the Lagrange multiplier of the borrowing constraint in (A.7). The result is trivial if $\lambda(a) > 0$. If $\lambda(a) = 0$, (C.1) can be written as:

$$\left(\frac{1}{1+\tau_b}\right)\left(\frac{W_o(a - (1+\tau_b)\,p) - W_r(a)}{m_b W_o'(a - (1+\tau_b)\,p)}\right) = \hat{p}'(m_b). \tag{C.2}$$

This equation has a unique solution (Proposition 1). The term in the left-hand side is the buyer's marginal rate of substitution of p for m_b. Buyers prefer high values of m_b and low values of p. Given the assumption on $g_r^c(a)$, W_o is strictly concave, by Proposition 12. If $(W_o(a - (1+\tau_b)\,p) - W_r(a))$

increases with a for a given p, this implies that the buyer's marginal rate of substitution is strictly increasing in a and, hence, so is the optimal value of m_b. $\square$

The proof of Proposition 3 is based on the original problem where potential buyers choose θ. The result follows from the continuity and differentiability of W_b and W_r, and Proposition 1.

Proof. Let $\tilde{W}_b(a)$ denote the value of a potential buyer conditional on participating in the afternoon market, that is, the value of problem (3.2). Let $\tilde{g}_b^\theta(a)$ be the associated policy function. Then

$$W_b(a) = \max\{\tilde{W}_b(a), W_r(a)\}, \tag{C.3}$$

and $\tilde{g}_b^\theta(a) = g_b^\theta(a)$ if $W_b(a) = \tilde{W}_b(a) > W_r(a)$. Since θ_0 is only feasible choice for a potential buyer when $a \leq a_{min} = (1+\tau_b)\, p_{min} - (1-\delta)\, \overline{p}$, $W_b(a) = W_r(a)$ on this range. Suppose $a > a_{min}$, so the constraint set of problem (3.2) is nonempty. Applying the Envelope theorem to the Lagrangian of this problem yields

$$\tilde{W}_b'(a) - W_r'(a) = m_b\left(\tilde{g}_b^\theta(a)\right)\left(W_o'\left(a - (1+\tau_b)\, p(\tilde{g}_b^\theta(a))\right) - W_r'(a)\right) + \lambda(a). \tag{C.4}$$

The righthand side of (C.4) is strictly positive because $m_b(\theta) > 0$ for all $\theta \in \mathbf{R}_+$, the term in brackets is strictly positive by assumption, and $\lambda(a) \geq 0$. Thus $\tilde{W}_b(a) - W_r(a)$ is strictly increasing for $a > a_{min}$. By assumption, $W_b(a) = \tilde{W}_b(a) > W_r(a)$ for some a. Since $\tilde{W}_b$ and W_r are continuous, there then exists a_{part} such that $W_b(a) = \tilde{W}_b(a) > W_r(a)$ for all $a > a_{part}$ and $W_b(a_{part}) = \tilde{W}_b(a_{part}) = W_r(a_{part})$. Since $p(g_b^\theta(a)) > p_{min}$ for $a > a_{part}$, $p(\theta)$ is continuous, and so is $g_b^\theta(a)$ on this range (by Proposition 3.2), $p(\lim_{a\to a_{part}^+} g_b^\theta(a_{part})) > p_{min}$. Thus $a_{part} > a_{min}$ and, by continuity, this inequality also holds for any $a < a_0$ sufficiently close to a_{part}. Since $\tilde{W}_b(a) - W_r(a)$ is strictly increasing on this range, $W_b(a) = W_r(a) > \tilde{W}_b(a)$ and so $g_b^\theta(a) = \{\theta_0\}$ for any $a < a_{part}$. $\square$

Finally, when the borrowing constraint holds for some buyers and is slack for others, the existence of the threshold a_1 follows directly from the following result, which uses the differentiability of W_o and W_r and the strict monotonicity of W_r.

Lemma 13. *If $a < a'$ and $\hat{\lambda}(a)$, $\hat{\lambda}(a') > 0$ then $\hat{\lambda}(a') < \hat{\lambda}(a)$.*

Proof. If $\hat{\lambda}(a) > 0$, the price paid by a buyer with assets a is $\frac{a+(1-\delta)\overline{p}}{(1+\tau_b)}$. Thus (C.1) implies

$$\begin{aligned}\hat{\lambda}(a) &= \frac{W_o\left(-(1-\delta)\,\overline{p}\right) - W_r(a)}{(1+\tau_b)\hat{p}'(m_b)} - m_b W_o'\left(-(1-\delta)\overline{p}\right) \\ &= \frac{W_o\left(-(1-\delta)\,\overline{p}\right) - W_r(a)}{(1+\tau_b)\hat{p}'(m_b)} - m_b\, u_c\left(g_o^c\left(-(1-\delta)\,\overline{p}\right), \hbar\right),\end{aligned} \tag{C.5}$$

where the last equality follows from the Envelope theorem. Also, since $\hat{p}(m_b)$ is given by (A.6), m_b satisfies

$$\frac{\kappa_s + (1-\beta)\overline{p}}{\hat{m}_s(m_b)} + \beta\overline{p} = \frac{a + (1-\delta)\overline{p}}{(1+\tau_b)}. \tag{C.6}$$

If assets increase from a to a' then m_b increases, since $\hat{m}_s$ is strictly decreasing, and so does $\hat{p}'(m_b)$, since $\hat{p}$ is strictly increasing and strictly convex. Since W_r is strictly increasing by Theorem 1, (C.5) then implies $\hat{\lambda}(a') < \hat{\lambda}(a)$. $\square$

D Computation

In order to compute a stationary equilibrium it is best to rewrite the problems of potential buyers and intermediaries so that, instead of choosing m_b taking $\hat{p}(m_b)$ as given, they choose p taking as given the inverse of the increasing function $\hat{p}(m_b)$, which we denote by $m_b(p)$. For the computation, it is crucial that $m_b(\theta)$ is a function instead of a correspondence. In particular, we cannot use the standard "truncated" Cobb-Douglas matching function. In our calibration, we use the class of matching functions in Chaumont and Shi (2018) (though the urn-ball matching function could also be used).

D.1 The matching function and the equilibrium price schedule

Given the Walrasian price $\overline{p}$, equation (A.6) determines m_s as a function of p:

$$m_s(p) = \frac{\kappa_s + (1-\beta)\,\overline{p}}{p - \beta\,\overline{p}}. \tag{D.1}$$

This function is strictly decreasing and strictly convex with $m_s(p_{min}) = 1$ and $\lim_{p\to\infty} m_s(p) = 0$, and does not depend on the choice of the matching technology.

We take $m_s(\theta) = (1+\theta^{-\gamma})^{\frac{-1}{\gamma}}$ with $\gamma > 0$, and $m_b(\theta) = m_s(\theta)/\theta$. Thus $\hat{m}_s(m_b) = \left(1 - m_b^{\gamma}\right)^{1/\gamma}$, and we can write

$$m_b(p) = \left(1 - m_s(p)^{\gamma}\right)^{1/\gamma}, \tag{D.2}$$

$$\theta(p) = \frac{m_s(p)}{\left(1 - m_s(p)^{\gamma}\right)^{1/\gamma}}. \tag{D.3}$$

Here, $\theta(p)$ is the inverse of $p(\theta)$, so it is strictly decreasing and strictly convex with $\lim_{p\to\infty}\theta(p) = 0$ and $\lim_{p\to p_{min}}\theta(p) = \infty$. Also, $m_b(p)$ is strictly increasing with $m_b(p_{min}) = 0$ and $\lim_{p\to\infty} m_b(p) = 1$. As shown in Appendix A, $m_b(p)$ is strictly concave provided $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is non decreasing. This last assumption can be further relaxed. For instance, for the value of γ used in our calibration to match the value of median time to buy in the data (and, in fact, for any $\gamma < 1$), the assumption only holds for values of m_b above some threshold. Yet we only require that it holds for the range of values of m_b which correspond the submarkets that are active in equilibrium (since eliminating inactive submarkets does not change the problem of a potential buyer). One can easily verify that it suffices to check that the slope of $-\hat{m}_s{}'(m_b)/\hat{m}_s(m_b)$ is positive for the lowest value of m_b observed in equilibrium (which corresponds to the optimal choice of a marginal buyer). If so, $m_b(p)$ is strictly concave on the range of prices which correspond to the set of active submarkets, and the results in Propositions 1 to 3 again hold.

D.2 The optimal choice of potential buyers

In order to extend the method in Fella (2014) to our framework, we proceed in two steps. The problem of those potential buyers who participate in the afternoon market in equilibrium can be written as

$$\begin{array}{ll} W_b(a) & \max_p \left\{ W_r(a) + m_b(p) \left[W_o\left(a - (1+\tau_b)\, p\right) - W_r(a) \right] \right\} \\ \text{s. t.} & p_{min} \leq p \leq \frac{a + (1-\delta)\,\overline{p}}{(1+\tau_b)}, \end{array} \tag{D.4}$$

with associated policy function $g^p(a)$. By Proposition 3, the constraint $p \geq p_{min}$ does not bind. The buyer's gains from trading at price $p > p_{min}$ are $S(a,p) = W_o\left(a - (1+\tau)p\right) - W_r(a)$.

By Theorem 1, $S(a,p)$ is strictly decreasing in p. Hence, if $S(a, p_{min}) \leq 0$ then $S(a,p) < 0$ for all $p > p_{min}$, and non-participation is optimal in this case. Suppose that $S(a, p_{min}) > 0$, so the gains from participation are positive. It is direct to check from the first-order condition of problem (D.4) that the Lagrange multiplier of the borrowing constraint is given by $\lambda(a) = m_b'(p)[S(a,p) - \tilde{S}(a,p)]$, where $\tilde{S}(a,p) = \frac{m_b(p)}{m_b'(p)} u_c\left(g_o^c\left(a - (1+\tau_b)\,p\right), \hbar\right)(1+\tau_b)$. Hence, at an optimal solution, $S(a,p) \geq \tilde{S}(a,p)$, with equality if the constraint does not bind. By the Envelope Theorem, $W_o' = u'\left(g_o^c\left(a - (1+\tau_b)\,p\right)\right)\left(a - (1+\tau_b)\,p\right)$, so $g_o^c(a)$ is non-decreasing if W_o is concave, since u is strictly concave. Since m_b is strictly increasing and strictly concave, this implies that $\tilde{S}(a,p)$ is strictly increasing in p and non-increasing in a. Also, $\tilde{S}\left(a, p_{min}\right) = 0$ regardless of the value of a, since $m_b\left(p_{min}\right)/m_b'\left(p_{min}\right) = 0$. There is then a unique value p which solves $S(a,p) = \tilde{S}(a,p)$ (in line with Proposition 1), and for this value $S(a,p) > 0$. There are then two cases: (i) if $p \leq \left(a + (1-\delta)\,\overline{p}\right)/(1+\tau_b)$ then $g_p(a) = p$, and (ii) otherwise, $g^p(a) = \left(a + (1-\delta)\,\overline{p}\right)/(1+\tau_b)$.

We use the following algorithm to find $g^p(a)$. Given the value functions W_o, W_r and the policy function g_o^c:

1. Check that $S\left(a, p_{min}\right) > 0$, so the agent's gains from participation are positive. (Otherwise, $g^\theta(a) = \theta_0$).
2. Find the maximum price the agent is willing to pay. This is equal to $p_r = \widetilde{p}$ where $S\left(a, \widetilde{p}\right) = 0$ if $\widetilde{p} \leq \left(a + (1-\delta)\,\overline{p}\right)/(1+\tau_b)$. Otherwise, $p_r = \left(a + (1-\delta)\,\overline{p}\right)/(1+\tau_b)$.
3. If $\tilde{S}\left(a, p_r\right) > S\left(a, p_r\right)$ use any solver to find a price $p \in \left(p_{min}, p_r\right)$ for which $\tilde{S}\left(a, p\right) = S\left(a, p\right)$.
4. If $\tilde{S}\left(a, p_r\right) \leq S\left(a, p_r\right)$, set $p = p_r$.

If $S(a,p)$ is increasing in a, as in our quantitative model, the above arguments imply that both p_r and $g^p(a)$ increase with a (in line with Proposition 2). Agents with low assets are constrained and choose $p = \left(a + (1-\delta)\,\overline{p}\right)/(1+\tau_b)$. Wealthier agents are unconstrained.

D.3 The choice of financial assets

Let us focus on the problem solved by the renter at night. The expression for the Euler equation of the problem depends on whether the agent can participate in the competitive search market in the next afternoon. Thus there are two cases. If $g_r^a(a) + (1-\delta)\,\overline{p} < (1+\tau_b)p_{min}$, the Euler equation is:

$$-u_c\left(g_r^c(a), h_r\right) + R\,\beta\, u_c\left(g_r^c(a'), h_r\right) \leq 0, \tag{D.5}$$

with equality if $a' = g_r^a(a) > 0$. If $g_r^a(a) + (1-\delta)\,\overline{p} \geq (1+\tau_b)p_{min}$, the Euler equation becomes

$$-u_c\left(g_r^c(a), h_r\right) + R\,\beta\,\left[m_b\left(g^p(a')\right)\, u_c\left(g_o^c\left(a' - (1+\tau_b)\,g^p(a')\right), \hbar\right) + \left(1 - m_b\left(g^p(a')\right)\right)\, u_c\left(g_r^c\left(a'\right), h_r\right)\right]$$
$$+ R\,\beta\,\frac{m_b'\left(g^p(a')\right)}{1+\tau_b}\,\left[S\left(a', g^p(a')\right) - \tilde{S}\left(a', g^p(a')\right)\right] \leq 0, \quad \text{(D.6)}$$

with equality if $a' = g_r^a(a) > 0$. The problem solved by owners is similar, except for the fact that they can borrow up to $(1-\delta)\,\overline{p}$. We build on Fella (2014) and solve for the optimal consumption rule using a modified version of his generalized endogenous grid method. The algorithm is as follows:

1. Choose an initial guess for $(W_o^j,\, W_r^j,\, g_o^{c,j},\, g_r^{c,j})$. For the owner's value function, we use the value function of an owner that is never hit by any shock as an initial guess. For the renter, we use that of a renter who never participates in the afternoon market. The consumption policy function of the renter will have a discontinuity point. We choose $\underline{a}^j = (1+\tau_b)\,p_{min}$ as the first guess for this point.

2. Solve the afternoon problem as outlined in Section D.2 to find $g^p(a)$ and $W_b(a)$.

3. For a given grid for next period's assets, ga, we use the Euler equation to find consumption today. We know that, if $ga < \underline{a}^j$, the Euler equation is (D.5); otherwise it is (D.6). We need to interpolate to obtain the consumption policy function as a function of the grid of assets today. We also need to be aware that there is a discontinuity at $\underline{a}^j$. This is key to use interpolation to find the policy function of consumption (as a function of assets today). To find the global maximum in a particular region, we follow Fella (2014). There is a cutoff point below which the renter knows that she will not participate in the afternoon market in next period. Save the node as $\underline{a}^{j+1}$. Save W_o^{j+1}, W_r^{j+1}, $g_o^{c,j+1}$, $g_r^{c,j+1}$.

4. Go to step 2. Iterate until convergence.

A grid of 400 points in financial assets gives very high accuracy and is very fast.

D.4 The stationary distribution

We cannot use Monte Carlo simulations in this setup because of the curse of dimensionality. Monte Carlo simulations are a good approximation of the invariant distribution when we are certain that the law of large numbers holds across simulations. This is not the case here, however. This is so because the level a of financial assets is a continuous variable. In this case, any change in the distribution of financial assets implies a change in the number of submarkets which are active in equilibrium. Using Monte Carlo simulations would require using a sample so large that the law of large numbers holds in each possible submarket, which is computationally unfeasible.

We thus solve for the stationary distribution as in Huggett (1993) and as explained in Ríos-Rull (1997). We use a much finer grid than the one used to solve the household's problem (800 points in our case) and guess the distribution of owners and renters at night. Then we use the policy functions for financial assets to integrate numerically and find the distribution of non-traders and potential buyers in the afternoon as shown in equations (2.11)–(2.14). We iterate until convergence.

D.5 The algorithm to find the stationary equilibrium

1. Choose an initial guess for the Walrasian price $\overline{p}$ and obtain the price function in (D.2).
2. Solve the household's afternoon problem as stated in Subsection D.3.
3. Find the invariant distribution. Calculate the mass N of non traders in the afternoon.
4. Given the stationary distribution of buyers, use (2.15) to calculate the density of buyers for each level of financial assets, $b(a)$. For the buyers who participate, use $g^p(a)$ to calculate the probabilities of selling and buying, and thus

$$\widetilde{\theta}(a) = \frac{m_s\left(g^p(a)\right)}{m_b\left(g^p(a)\right)}. \tag{D.7}$$

5. Find the amount of vacant homes needed to satisfy the rational expectations condition at the guessed prices:

$$S' = \int_A \frac{b(a)}{\widetilde{\theta}(a)}\, da. \tag{D.8}$$

6. Compare S' with the actual number of vacant homes, $H - N$. If $S' > H - N$ (the price $\overline{p}$ is too low), update $\overline{p}$ upwards. If $S' < H - N$, update $\overline{p}$ downwards. Go back to step 1.

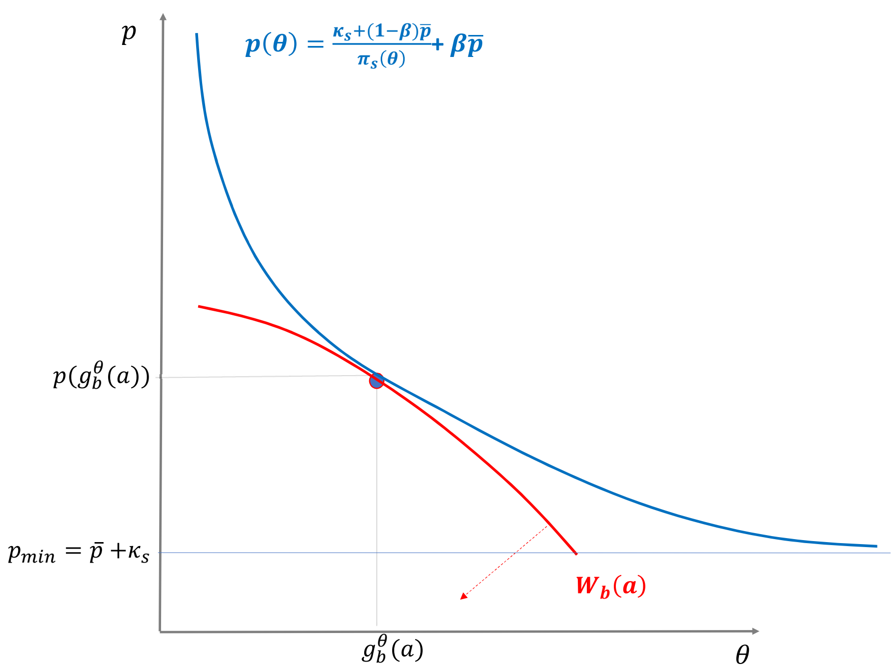


Figure 1: The optimal choice of an unconstrained buyer.

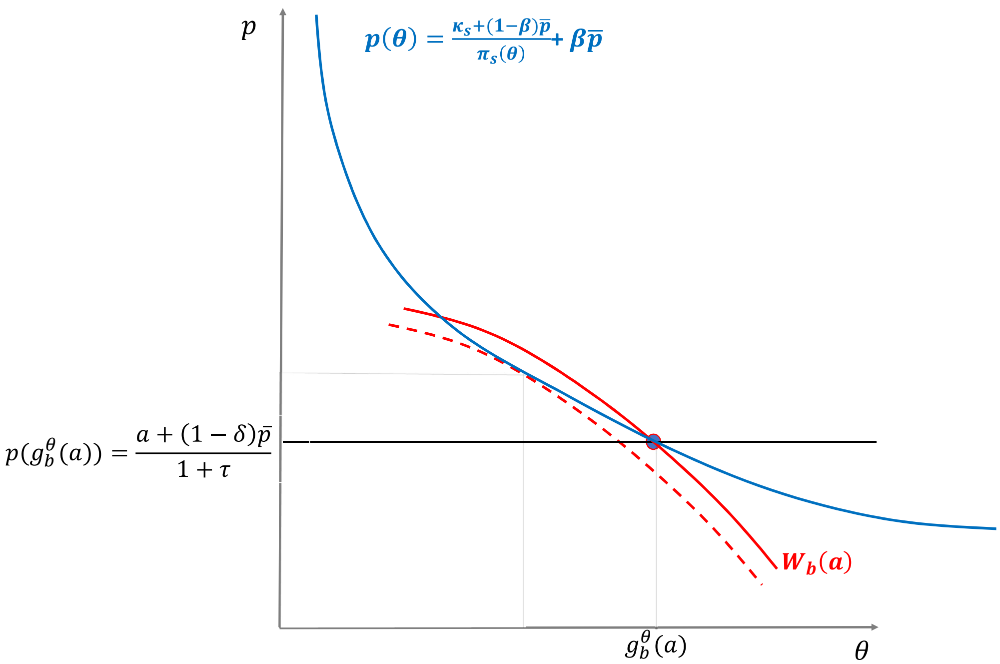


Figure 2: The optimal choice of a borrowing-constrained buyer.

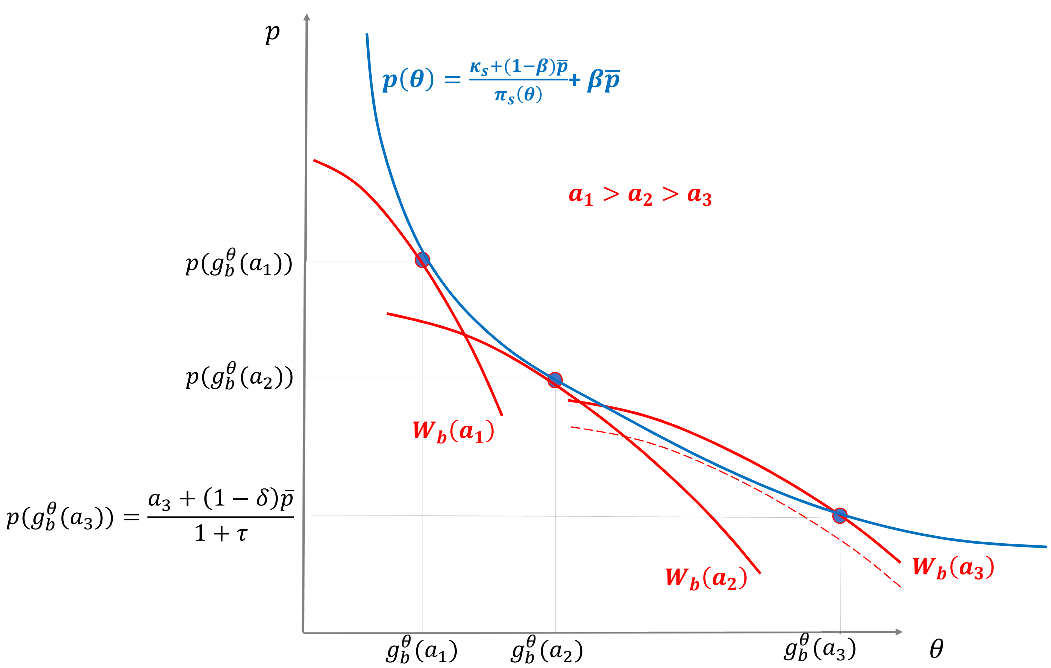
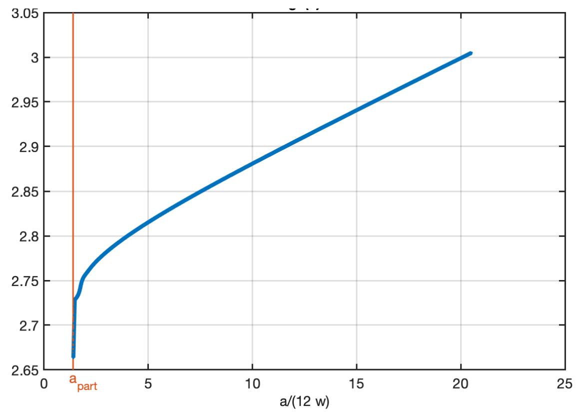
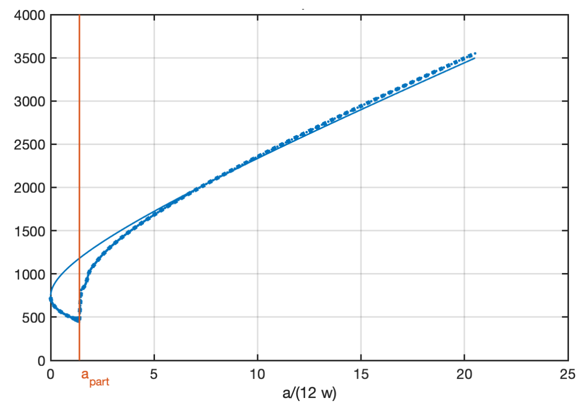


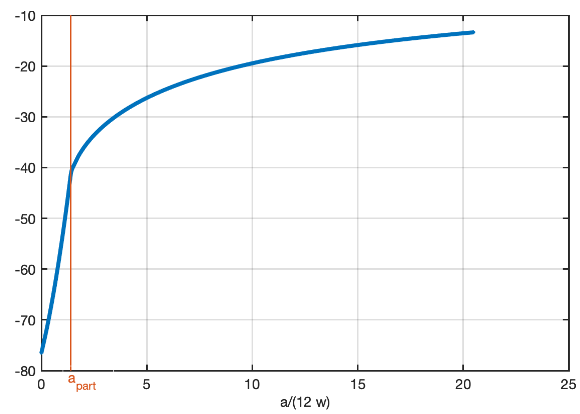
Figure 3: Positive sorting.



(a) The price policy function $g^p(a)$

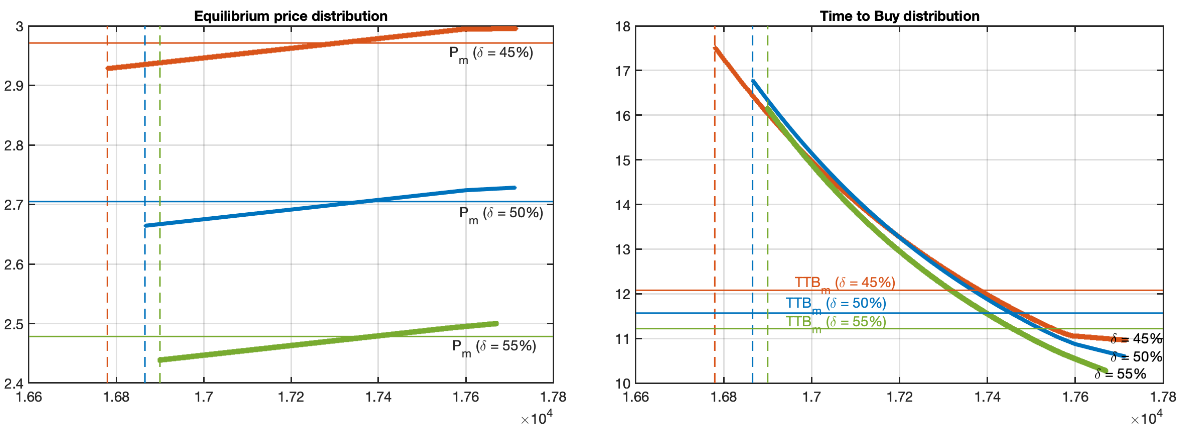


(b)(c) The renter's consumption policy function $g_r^c(a)$ Probability of buying as a function of financial assets $m_b(g^p(a))$



(d) The renter's value function $W_r(a)$

Figure 4: Policy functions



(a) (b)

Figure 5: Prices and time to buy as a function of financial assets for different values of δ.

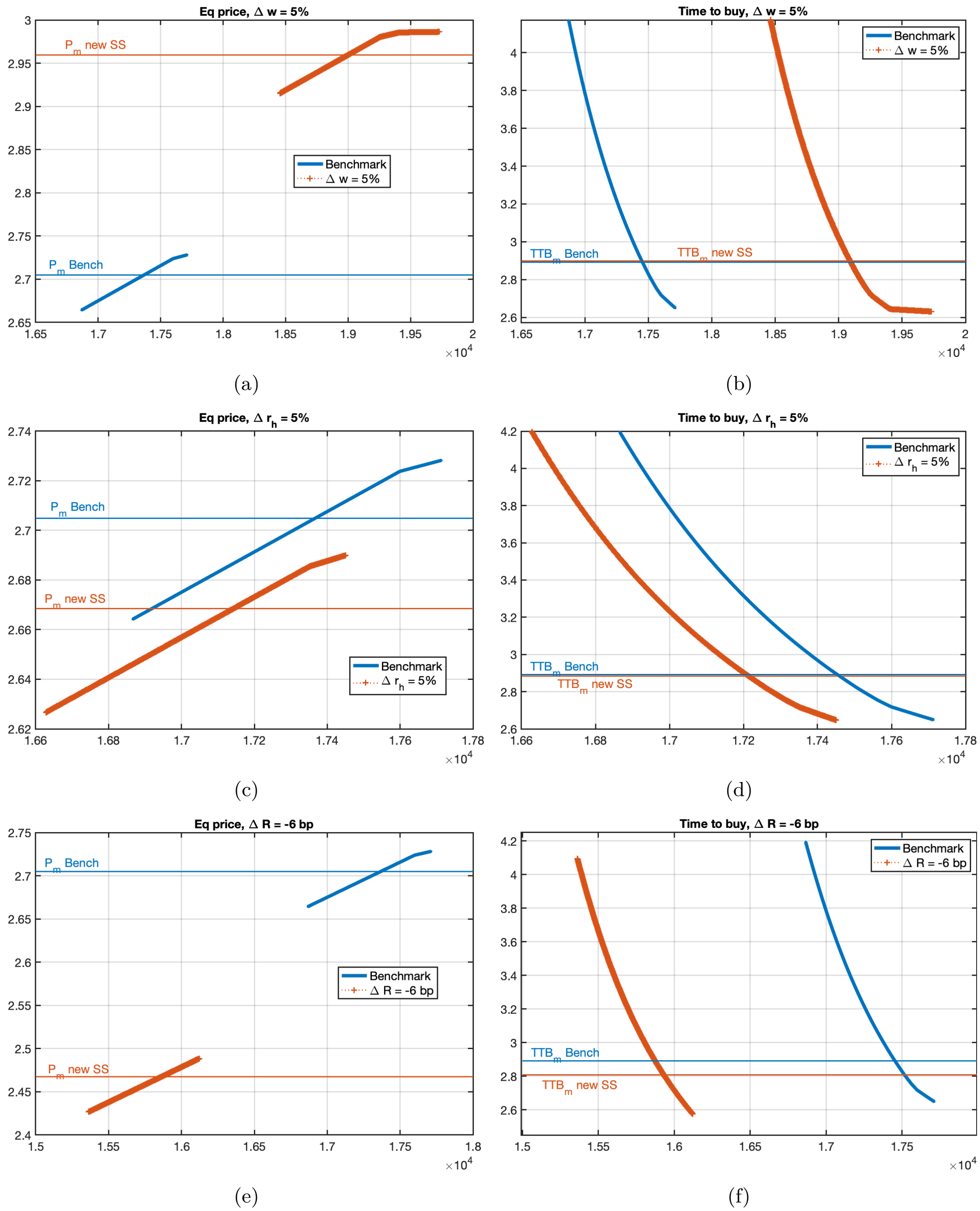


Figure 6: Prices and Time to Buy as a function of financial assets (I)

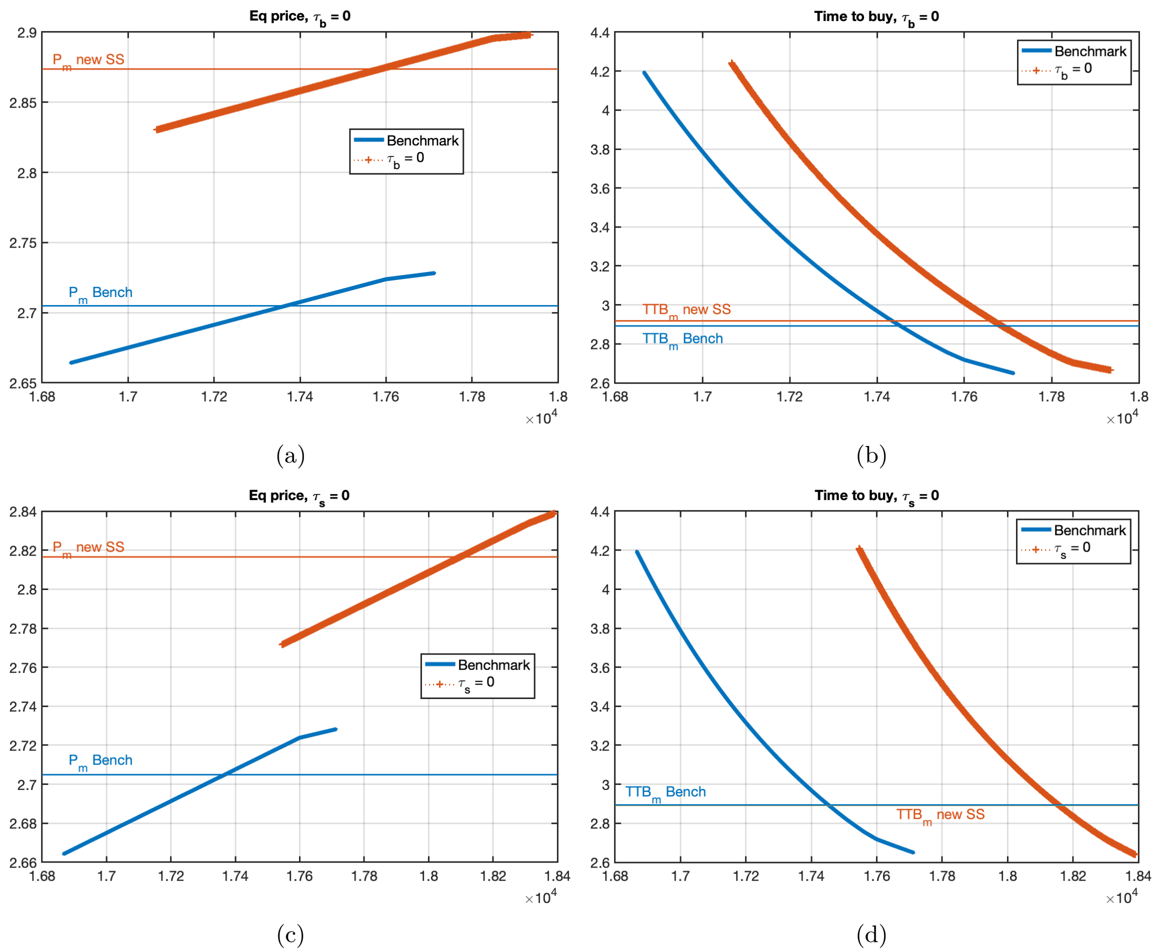


Figure 7: Prices and Time to Buy as a function of financial assets (II).

Table 1: Calibration

Param.	Observation	Value
w	Monthly wage	1000.0000
r (annually)	Díaz & Luengo-Prado (IER 2010)	0.0391
r_h	AHS, median housing costs renters 28% of income	$0.25\,w$
τ_b	Indirect taxes on buyers	0.0250
τ_s	Indirect taxes on owners	0.0600
κ_s	Cost of posting a vacancy	0.0000
δ	SCF, median LTV ratio = 41.04%	0.5000
π_μ	NAR: Median tenure of 10 years	0.0059
π_{ξ_o}	Annual mobility of owners = 3.2 %	0.0025
π_{ξ_r}	Annual mobility of renters = 12 %	0.0100
σ	Risk aversion parameter	2.0000
$h_r/\hbar$	Homeownership rate = 69.43%	0.9996
β	Median W/E for renters = 0.3450	0.8480
γ	Median Time To Buy (NAR) [10 12]	0.6552
H/N (%)	Median H/E for owners = 2.7223	70.3789

Notes: The model period is a month. Annualized values. The monthly wage w is the numeraire and has been set $w = 1000$.

Table 2: The benchmark steady state

Target	Benchmark	Data	Source
Homeownership rate	69.4280	69.4258	SCF mean1989-2007
Median H/E owners	2.7182	2.7223	SCF median 1989-2007
Median LTV ratio	38.6840	41.0448	"
Median W/E renters	0.2819	0.3450	"
Median Time to Buy	11.5670	[10-12]	NAR 2017
Rent-to-Price ratio (%)	9.4319	[8-15]	Sommer and Sullivan (2018)
Median TOM	10.2620	[4-17]	NAR 2017
Months of Supply	2.6068	5.4737	NAR 2017
Vacancy rate (%)	2.1816	2.1766	AHS, mean 2011-2015
C.V. of prices (%)	0.9269		
Mean error (%)	0.8489		
% Sales out of 1% price interval	59.2480	49.8%	Zillow 5% interval
Gini of renters wealth G_r	0.5345		
Gini of wealth (all) G	0.2221		

Notes: Median TTB refers to median Time to Buy, whereas media Time on the Market refers to time to sell. Both statistics are reported in weeks. The rest of the statistics are reported in annual terms.

Table 3: Long run changes in the down payment

Target	$\delta = 0.45$		$\delta = 0.5$	$\delta = 0.55$	
	Fixed H	Const.	Benchmark	Fixed H	Const.
$\overline{p}/\overline{p}_{bench}$	1.0996		1.0000	0.91491	
H/H_{bench}		0.9111	1.0000		-
Homeownership rate	69.4850	76.2090	69.4280	69.3790	-
Median H/E owners	2.9712	2.7182	2.7049	2.4782	-
Median LTV ratio	43.8840	43.5800	38.6840	33.7430	-
Median W/E renters	0.2910	0.4007	0.2819	0.2731	0.0058
Rent-to-Price ratio (%)	8.5773	9.4340	9.4319	10.3090	9.4340
Median Time to Buy	12.0780	11.5920	11.5670	11.2200	-
Median TOM	9.8880	10.2000	10.2620	10.5950	-
Months of Supply	2.5078	2.6012	2.6068	2.6890	-
Vacancy rate (%)	2.1019	2.3837	2.1816	2.2509	0.0000
CV of prices (%)	0.5065	0.7409	0.9295	1.0244	-
% of sales error > 1%	42.9450	43.5820	59.2480	47.7330	-
Price range	1.0752	1.0244	1.0241	1.0251	-
Gini of renters wealth	0.5275	0.4572	0.5345	0.5381	0.0000
Gini of wealth (all)	0.2220	0.1622	0.2221	0.2222	0.0000
Potential buyers	31.1000	24.4330	31.1560	31.2050	100.0000
Participation rate	6.8527	9.3561	6.6836	6.4148	0.0000
Marg. buyer $a/(12\,w)$	1.3983	1.2713	1.4055	1.4083	1.5390
Actual purchases (%)	27.6780	28.2130	28.2850	29.4310	-

Table 4: Steady state comparisons

Target	Bench.	$\Delta w =$ 5%		$\Delta r_h =$ 5%		$\tau_b = 0$		$\tau_s = 0$		$\Delta R =$ $-6bp$	
		Fixed H	Const.	Fixed H	Const.	Fixed H	Const.	Fixed H	Const.	Fixed H	Const.
$\overline{p}$	1.0000	1.0942		0.9859		1.0625		1.0403		0.9109	
H/H_{bench}	1.0000		1.0598		-		1.0621		1.0641		-
Homeownership rate	69.4280	69.4350	73.5390	69.4210	-	69.4390	73.7200	69.4220	73.8520	70.0410	-
Median H/E owners	2.7182	2.6844	2.4555	2.4204	-	2.8737	2.7059	2.8165	2.7075	2.4674	-
Median LTV ratio	38.6840	38.9450	38.6430	38.6600	-	41.2340	41.2050	44.5490	44.5350	38.6280	-
Median W/E renters	0.2819	0.2912	0.3335	0.2521	0.0036	0.2819	0.3544	0.2873	0.3734	0.2359	0.0036
Rent-to-Price ratio (%)	9.4319	8.6198	9.4319	10.0450	9.9035	9.3208	9.4319	9.5195	9.4319	10.8720	9.4319
Median Time to Buy	11.5670	11.5930	11.2990	11.5360	-	11.6720	11.5020	11.5760	11.4920	11.2350	-
Median TOM	10.2620	10.2060	10.4010	10.3120	-	10.1920	10.2870	10.3110	10.3690	10.5180	-
Months of Supply	2.6068	2.5955	2.6800	2.6196	-	2.5899	2.6454	2.6208	2.6541	2.6896	-
Vacancy rate (%)	2.1816	2.1721	2.3666	2.1917	-	2.1664	2.3421	2.1896	2.3587	2.2622	-
CV of prices (%)	0.9269	0.9235	0.6679	0.9115	-	0.8728	0.6499	0.7908	0.7686	0.6667	-
% of sales error $> 1\%$	59.2480	39.2010	37.5070	59.0620	-	58.2730	34.1980	44.4890	44.2440	36.4160	-
Price range	1.0241	1.0244	1.0257	1.0241	-	1.0239	1.0248	1.0242	1.0248	1.0255	-
Gini of renters wealth	0.5345	0.5274	0.4862	0.5349	0.0000	0.5337	0.4831	0.5350	0.4800	0.5549	0.0000
Gini of wealth (all)	0.2221	0.2206	0.1843	0.2222	0.0000	0.2207	0.1806	0.2098	0.1679	0.2219	0.0000
Potential buyers	31.1560	31.1500	27.0800	31.1640	100.0000	31.1460	26.9000	31.1620	26.7700	30.5490	100.0000
Participation rate	6.6836	6.7342	7.8089	6.6375	0.0000	6.7242	7.9828	6.5693	8.0085	6.5492	0.0000
Marginal buyer $a/(12w)$	1.4055	1.3949	1.2745	1.2569	1.4052	1.4222	1.3382	1.4622	1.4052	1.2802	1.4052
Actual purchases (%)	28.2850	28.0780	29.3900	28.4660	-	28.1110	29.0160	28.7230	29.1730	29.5870	-

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